ARIADNE'S THREAD, DAEDALUS' WINGS, AND THE LEARNER'S AUTONOMY

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Résumé :Il existe une tension apparente entre l'idée d'apprenant autonome, promue par les tenants des classes « participatives », et l'argument participatif selon lequel l'apprentissage est un processus intrinsèquement collectif d'induction vers des formes d'action historiquement établies. Dans cet article, nous essayons de comprendre la nature de cette tension et ses conséquences pour les pratiques d'éducation. Nous commençons par une brève présentation de la perspective commognitive sur l'apprentissage, qu'on peut considérer comme une version particulière de l'approche participative, selon laquelle la pensée est une forme individualisée de la communication interpersonnelle et l'apprentissage scolaire un processus de modification et d'extension de ce discours. Nous introduisons alors la distinction entre apprentissage niveau-objet et apprentissage niveau-meta, ce dernier nécessitant de suivre ceux-qui-savent plutôt que de se limiter à conduire ses propres explorations inventives. Nous soutenons qu'un certain taux de compréhension et d'accords mutuels est nécessaire pour que l'apprentissage méta soit efficace. Quelques exemples illustratifs, pris en classes de mathématiques et de statistiques, montrent un apprentissage qui survient alors que l'accord est respecté, et d'autres ce qui se passent lorsque certains éléments de cet accord sont violés. Nous concluons avec une mise en garde contre une interprétation rapide et unidimensionnelle du principe de l'autonomie de l'apprenant.

Key-Words : *Autonomie de l'apprenant, discours, apprentissage niveau objet, apprentissage niveau méta, accord d'enseignement apprentissage, commognition*

Introduction: Learning and Autonomy

For humans, every interaction constitutes an opportunity for both learning and teaching. The overpowering tendency toward educating and being educated is a natural entailment of our being inherently social creatures whose whole existence depends on their capacity for collective action. To ensure interpersonal coordination, individuals need to adhere to certain routines and mitigate individual deviations. This can only be done by constant mutual adjustment and fine-tuning.

Encounters between novices and experts are probably the most obvious, indeed irresistible, invitations to teaching and learning, and school is the place that brings novices and experts together and institutionalizes their roles as, respectively, learners and teachers. This article is an extended reflection on the division of labor between those who learn and those who teach. More specifically, we focus on the question of how autonomous one can be as a learner when her spontaneously developed ways of acting are thrown into the melting pot of schooling.

There are probably as many possible variations on the theme of learner's autonomy as there are people who learn. Just think about the striking contrast between the learning processes induced by the mythological heroes Daedalus and Ariadne when each one of them was trying to help his or her loved-ones to escape King Minos' prisons. Ariadne, to guide her beloved Theseus through Minotaur's labyrinth, provided the young man with a thread which he was told to follow faithfully and without questions. Daedalus, on the other hand, armed his son Icarus with wings and let him choose his own trajectory. These two instructional approaches may well be marking the opposite ends of the spectrum of possibilities available to teachers who wish to lead uninitiated through the mazes of mathematics, history or literature. The question of which option should be chosen is far from simple. Although Ariadne's predesigned learning trajectory seems to be the surest path toward the teacher's goal [1], her approach would probably be criticized by those for whom the principle of honoring the learner's own thinking is not any less important than the final outcome of her explorations. The most radical interpreters of the idea of student's autonomy are likely to adopt Daedalus's approach. Alas, one should not forget the unhappy ending of Icarus' story [2]. Obviously, there is no one fully satisfactory answer to the question of what it means for the learner to be autonomous.

This said, at any given time in history some pedagogies are more popular than some others. Ariadne's prescriptive tactics, although rather extreme, is not too distant from what was actually happening in schools for ages, until just a few decades ago. The risks inherent in the Daedalus' choice notwithstanding, there is a clear shift nowadays toward letting students choose their own learning trajectories. This preference can clearly be felt in many policy documents, and the one called *Principles and Standards for school mathematics* seems to be among them:

> A major goal of school mathematics programs is to create *autonomous learners*… Students learn more and learn better when they *take control* of their learning by defining their goals and monitoring their progress. When challenged with appropriately chosen tasks, students become confident in their ability to tackle difficult problems, eager to *figure things out on their own*, flexible in exploring mathematical ideas and trying alternative solution paths, and willing to persevere (National Council of Teachers of Mathematics, 2000, p. 21; emphases added).

On the face of it, these principles are indeed very much in tune with Daedalus' pedagogy: the request for autonomy in learning seems to imply that the student should be the principal, if not the only, designer and implementer of the learning process. At a closer look, however, the idea of autonomy admits of several interpretations. How one translates such terms as *control of learning* or *figuring things out on their own* into actions depends on his or her understanding of the term *learning*. Only those who think of learning as "acquiring knowledge" and of knowledge itself as a portable entity that resides somewhere in the world and can be reproduced by the student within her head are likely to view the above exhortation as supporting Daedalus' decisions. In this *acquisitionist* language, learning implies a conversation with the world rather than with other people, and the phrase "learner's own control of learning" can, indeed, be understood as implying student's almost unbounded freedom in deciding how and where new knowledge can be found ("figured out").

The interpretation changes rather dramatically when one switches to a *participationist* perspective that defines learning as initiation to patterned, historically established forms of activity. Being inherently social in their origins, these uniquely human forms of doing cannot possibly be mastered without an interaction with a competent doer and without a genuine wish to follow in the expert's footsteps. Although one can still talk about learner's autonomy, the terms "control of learning" and "figuring on one's own" must now be interpreted as referring to one's collaboration with people rather than directly with the world as such. More specifically, the "control" is to be understood as a form of command over different forms of interaction with others, and the "sense-making" is to be interpreted as student's effort to make sense of *foreign forms of talk* about the world rather than trying to fathom the nature of this world in a direct manner.

There is an apparent tension between the notion of autonomous learning and the idea of learning as an inherently collective process of induction to historically established forms of action. The wish to understand the nature of this tension and its consequences for educational practice fuels our efforts along the following pages. Our guiding assumption is that learner's autonomy is possible and desirable also when learning means adopting other people's ways of acting. We begin with a brief presentation of the commognitive perspective on learning, a particular version of the participationist approach, according to which thinking is an individualized form of interpersonal communication and school learning is a process of modifying and extending one's discourse. We then introduce the distinction between object-level and meta-level learning, of which only the latter requires trying to follow those in-the-know rather than just conducting one's own inventive explorations. We claim that a certain amount of mutual understandings and agreement is necessary if the meta-learning is to be effective (that is, if it is to bring the change expected by the teacher and the curriculum developer). A number of examples, coming from mathematics and statistics classrooms, will show a case of learning occurring when the agreement is respected, and will instantiate what happens when its different elements are violated. We conclude with a caveat against careless, one-dimensional interpretation of the principle of learner's autonomy.

Commognitive Perspective on Learning

Basic commognitive tenets: Thinking as individualized communication and school learning as becoming adept in historically established discourses.

Commognitive perspective on learning (Sfard, 2007, in press) is rooted in the participationist assumption that all uniquely human skills are products of individualization of historically established collective activities. Thus, young children develop the ability to speak, read or cook by gradually turning from "legitimate peripheral participants" (Lave & Wenger, 1991) who can only implement small parts of the job in collaboration with others into independent performers, who can do the task on their own and resort to it on their on accord. Thinking, probably the most distinctive of human forms of doing, is no different: it too is developmentally secondary to a certain patterned collective form of doing, namely, to the activity of communicating. According to the commognitive assumption, uniquely human form of thinking appears when a child becomes able to communicate with herself the way others communicate with her. The word *commognition*, a combination of *cognition* and *communication*, was coined to epitomize this claim, that is, to always remind us that human thinking develops, both historically and ontogenetically, through individualization of interpersonal communication. This communication does not have to be verbal or audible. Within commognitive perspective, therefore, cognitive processes and processes of inter-personal communicating are but different manifestations of basically the same phenomenon.

Just as there is a multitude of recreational games – chess, bridge, basketball, etc. – played with diverse tools and according to diverse rules, so there are many types of commognition, differing one from another in their patterns, objects, and the types of mediators used. Like in the case of games, individuals may be able to participate in certain types of communicational activity and be unable to take part in some others. In this paper, the different types of communication that bring some people together while excluding some others will be called *discourses*. The diverse domains of human knowledge learned in schools – mathematics, physics, statistics, history, etc. – can now be seen as special types of communication. These different discourses are made distinct, among others, by their *vocabularies* (keywords and their use), *visual mediators*, *routines* and the *narratives* which the discursants tell and endorse (for a detailed presentation of these four characteristics see Sfard, 2007, in Press; Sfard & Lavie, 2005). While engaging in different forms of communication, people are telling different, mutually complementing and sometimes incommensurable stories about the world.

Given this definition of discourse, any human society may be divided into partially overlapping *communities of discourses*. To be members of the same discourse community, individuals do not have to face one another and do not need to actually communicate. The membership in the wider community of discourse is won through participation in communicational activities of any collective that practices this discourse, be this collective as small as it may. Learning mathematics, history or statistics may now be defined as individualizing respective discourses, that is, as the process of becoming able and willing, whenever appropriate, to have mathematical, historical or statistical communication not only with others, but also with oneself.

Levels of discursive learning

Having defined school-type learning as an activity in which the student modifies and extends her discursive repertoire, we may now distinguish between two levels of learning, object-level and meta-level, both of them necessary if a person is to gain satisfactory mastery of, say, mathematics, statistics or history. Let us explain this distinction with the help of examples.

The main goal of a person who engages in a discourse learned in school is to become more knowledgeable about the objects of this discourse. Thus, while learning zoology, students get acquainted with different types of animals, while studying history they learn about past communities, and while engaging in mathematical discourse they investigate such mathematical objects as numbers, triangles, sets and functions. The explorations, which eventually produce endorsed narratives about the object in question, are usually done according to well defined meta-rules[3], specific to the given type of discourse.

The rather straightforward type of learning presented in the above paragraph will from now on will be called *object-level*. This is not the only one that is required, though, if the discourse is to evolve in accord with the historical development of mathematical communication. One developmental phenomenon that is quite general but is particularly salient in scientific, mathematical, and statistical discourses is an occasional *emergence of new objects of investigation*. Indeed, some of the "things" that are investigated by scientists or mathematicians, rather than being found directly in the world, are produced through the discourse itself. Thus, notions such as *velocity* and *energy* in physics, *distribution* and *mean* in statistics, or *set* and *function* in mathematics, although clearly related to observable real-world phenomena are, in fact, discursive constructs created for the sake of a better description of reality[4]. Together with new objects, the meta-rules of investigation and endorsement may also change, sometimes beyond recognition (Sfard, 2007). For example, when mathematical discourse is extended so as to include negative numbers, the meta-rule according to which one endorses basic statements about mathematical objects (e.g., "2+3=5") on the basis of extra-discursive evidence is no longer in force: the only argument one can use to substantiate the claim that 'minus times minus is plus' is that of the inner consistency and the usefulness of the thus created discourse. Learning that involves a change in meta-rules results in a discourse that is *incommensurable* with the one from which it evolved: some endorsed narratives of the old and new discourses may now sound contradictory and mutually exclusive, whereas in fact the respective discursants are simply using the same words in differing ways and judging endorsability according to different meta-rules[5]. Learning that results in an incommensurable discourse will be called *meta-level*[6].

Ways to learn

Whereas object-level learning leads simply to an extension of a discourse – it increases the set of "known facts" (endorsed narratives) about the investigated objects – meta-level learning is a transformation of the discourse: it changes the vocabulary and the ways in which explorations are done. There is, therefore, a significant difference in the nature of discursive change induced by the two types of learning: In mathematics, more than in any other discourse, the object-level change is a product of the logical necessity, whereas the meta-level transformation is a result of a historically sanctioned custom and is thus contingent rather than inevitable (Sfard, 2007). Because of this all-important difference, object-level and meta-level learning challenge the student in different ways and must thus follow significantly different paths.

Object-level learning occurs when the student is already reasonably familiar with the objects and meta-rules of the given discourse. At this point, the goal of the learning is to get better acquainted with the properties of the object. In mathematics, it means extensive *explorations* in which the known meta-rules are systematically applied. This, indeed, is what happens when one investigates, for instance, functions or system of equations, examines properties of numbers or of geometric figures, etc. This type of learning results, on the one hand, in new mathematical narratives which, after being endorsed are called *theorems*; and on the other hand, in a greater discursive proficiency of the student.

Meta-level learning is not nearly as straightforward. When faced with the need to deal with unfamiliar objects or with new meta-rules, the student confronts paradoxical, mutually contradicting requirements: since mathematical objects are discursive constructs, they can only arise by being talked about them; however, how can a person talk about an object without being already familiar with it? Similarly, the only possible reason to change meta-rules that have been working well in the past is the fact that the resulting discourse is more useful than the former; but how can the student become aware of the usefulness of the modified discourse without actually participating in it?

Because of this, and because of the contingency of meta-level transformations, the only conceivable way to induce the required meta-level changes is by letting the student immerse herself in a conversation with experienced discursant. More specifically, meta-level learning can only happen in the process of *scaf-*

folded individualization: the student joins experienced discursants in implementing discursive tasks, acting first only as a spectator and then as peripheral participant. At this point, the student experiences the new forms of talk as a *discourse-for-others* – as a mode of communication that she is ready to use in conversation with others because it makes sense to the others (this, as opposed to its being used in self-communication). As time goes by, the amount of the scaffolding required by the newcomer gradually decreases and her part in the implementation of relevant tasks steadily grows. If the process of individualization proceeds properly, the student, while becoming more and more independent as tasks' implementer, gradually *rationalizes* the new discourse, that is, discovers its inner coherence and becomes aware of its applicability and usefulness. This is, indeed, what needs to happen if the process of learning is to attain its ultimate goal of turning the discourse of others into a *discourse-for-oneself* – a discourse to which, from now on, the learner would turn spontaneously whenever it may help her in solving her own problems [7]. At this stage, the new form of communication turns into the discourse of one's thinking and the student becomes able to engage in independent object-level learning. This latter form of learning involves further explorations of discursive objects. This time, these latter routines may already be performed on the person's own accord and without any scaffolding. In the next section, we address the question of what needs to happen in order for such full cycle of meta-learning to be completed.

Meta-level Learning, Scaffolded Participation, and Learning-Teaching Agreement

Learning-teaching agreement as a condition for meta-level learning

Contrary to what might have been understood from the last paragraph, meta-level learning and object-level learning do not take place in a linear order, one after another. Because of the inherent circularities of the learning process, meta-level learning can only occur through scaffolded attempts at object-level explorations, albeit of a new kind. This means that the first step toward meta-level learning is an active engagement in a discourse that, at this initial point, is incommensurable with one's own. It is reasonable to assume that some special conditions must be fulfilled to ensure that the inevitable communicational gap between the learner and her more experienced interlocutors – teachers or more competent peers – catalyzes the desirable change rather than remaining an insurmountable hurdle to meaningful conversation. We now claim that the required change of the newcomers' discourse seems unlikely to occur without a *learning-teaching agreement* – without a certain set of unwritten understandings about those aspects of this learning process that are essential to its success. For the meta-learning to happen, all the participants need to be unanimous, if only tacitly, about at least three basic aspects of the communicational process: the *leading discourse*, their own respective *roles*, and the *nature of the expected change*. Let us say a few words on each of these requirements [8].

Agreement on the leading discourse. Interlocutors' consent to follow a more or less uniform set of discursive routines is the condition for effective communication. Although this agreed set of rules will usually be negotiated by the participants and will end up being probably somehow different from any of those with which each individual entered the interaction, the process of change may be ineffective if the interlocutors do not agree about which of these initial discourses should be regarded as setting the standards. The issue of leadership in discourse is, of course, a matter of power relations. In a traditional classroom, the power structure was supposed to be fully determined by the institutional context: the teacher was the leader by default, and open resistance counted as deviation from the norm. In a reform classroom, as defined, for example, in the NCTM policy document, *Principles and Standards* (NCTM, 2000), the issue of discursive leadership is open to negotiation, at least in principle.

Agreement on the roles of interlocutors. In addition to the agreement with regard to the model-discourse, those consensually recognized as leaders must be willing to play the role of teachers, whereas those whose discourses require adaptation must agree to act as learners. The acceptance of roles is not a formal act. Rather than expressing itself in any explicit declaration, this role-taking means a genuine commitment to the communicational rapprochement. Such agreement implies that those who agreed to be teachers feel responsible for the change in students' discourse and those who agreed to learn show confidence in the leader's guidance and are genuinely willing to follow in the expert participants' discursive footsteps (as documented in research literature, cases of student's resistance are not infrequent; see for example, Litowitz, 1997; Forman & Ansell, 2002). It is important to stress that this acceptance of another person's leadership does not mean readiness for mindless imitation. Rather, it means a genuine interest in the new discourse and a strong will to turn the new discourse from the discourse-for-others into a discourse-for-oneself.

Agreement on the necessary course of the discursive change. Students' persistent participation in mathematical talk when this kind of communication is for them but a discourse-for-others seems to be an inevitable stage in learning mathematics. If learning is to succeed, all the participants, the students and the teachers, have to have a realistic vision of what can be expected to happen in the classroom. In particular, all the parties to the learning process need to agree to live with the fact that the new discourse will initially be seen by the participating students as somehow foreign, and that it will be practiced only because of its being a discourse that others use and appreciate. To turn the discourse-for-others into a discourse-for-oneself, the student must actively explore other people's reasons for engaging in this discourse. This process of thoughtful imitation seems to be the most natural, indeed, the only imaginable way to enter new discourses[9]. It is driven by a need to communicate, so strong that it would often lead to what may seem in the eyes of some educators as the reversal of the "proper" order of learning: The learners would employ a rule enacted by another interlocutor as a prelude to, rather than a result of, their attempts to figure out the inner logic of this interlocutor's discourse. Without the overpowering urge to communicate and the resulting readiness for the thoughtful imitation, we might never be able to learn anything that is uniquely human – not even our first language.

Meta-level learning and learning-teaching agreement in collaborative learning –examples

With participatory classrooms being a relatively new phenomenon, we are still in the dark about many aspects of today's school learning. The dynamics of discursive lead-taking and lead-following is one of the most important topics for investigation. It is of particular interest to both theoreticians and practitioners in the case of collaborative learning. Indeed, the special feature of the collaborative setting is that both parties to the learning-teaching agreement are the students themselves and that there is no designated leader and no predetermined division of labor (e.g., Johnson & Johnson, 2006). In what follows, we explore four classroom episodes, trying to get a better sense of the relation between the quality of a learning-teaching agreement among the children and the effectiveness of meta-level learning.

More specifically, in each one of the four scenes we

(a) identify aspects of the discourse that imply that meta-level learning is necessary if the student is to make the transition to the new discourse;
(b) examine the interaction for the occurrence and the quality of learning-teaching agreement; and

(c) formulate a conjecture about the effectiveness of the learning that is taking place.

It is important to stress that because of the scarcity of the empirical material that can be actually presented, we will not be able to make decisive claims about the observed phenomena. Whatever answers will be given to the above three questions should be treated as but conjectures which, in order to be anything more than that, would require much additional evidence. In what follows, our intention is simply to illustrate the method of analysis that allows to identify different aspects of meta-level learning and of learning-teaching agreement, and which is therefore necessary to answer questions a, b and c, as formulated earlier.

Example 1: When learning-teaching agreement is in place and the leading discourse adheres to the established rules [10].

The vignette below is taken from a study in which a class of 12-year old seventh graders engaged, for the first time, in a discourse on negative numbers. The teacher, rather than presenting the students with ready made rules of operating on the extended number set, introduced several models (number line, arrow model, magic cubes [11]) and invited the students to raise their own conjectures about how the new numbers can be added, subtracted and multiplied. In the present episode, two students, Sophie and Adva, try to figure out how to multiply two numbers, one of which is positive (marked with a plus) and the other is negative (marked with a minus). The girls help themselves with the number line, appearing on the page in front of them.

Episode 1: Sophie and Adva look for the product of differently signed numbers

[1]	Sophie	Plus two times minus five [12] …	Points to the expression (+2)×(-5) written on the worksheet.
[2]	Adva	Two minus negative five	Shifts the worksheet closer to her and takes the pencil from Sophie.
[3]	Sophie	A, wait, hold on … plus two … it's as if you said minus five two times	
[4]	Adva	And what about plus two? What about the plus?	
[5]	Sophie	Minus five …	Looks intensely at what is written in front of her and speaks in «thinking aloud» mode.
		One, tow, three, four, five ..	Counts on the number line in front of her the unit segments to the left of the zero; marks -5.
		Times two – you know that plus two is two and you can drop the minus, right? So this is, as if …two multiplied by minus five, two times minus five ..	
		so it is minus five plus minus five	Turns to Adva and looks at her.
		It makes minus ten	
[6]	Adva	I don't know … the two plus here, perhaps the plus does mean something	Points at the plus sign before the 2.
[7]	Sophie	Right, you can drop the plus	
[8]	Adva	So this is like … you can drop the minus	
[9]	Sophie	No, not the minus, because this is as if you do two times minus five	
[10]	Adva	Fine, we shall do it and then see	
		Six times minus four … six times minus four	Reads the next expression, 6(-4) from the worksheet.
[11]	Sophie	Six times minus four … you do six times minus four, right … so it's minus twenty four	Writes -24 next to the equal sign on the worksheet.
[12]	Adva	Fine. «Write a question..»	Starts reading a new task from the worksheet.
[13]	Sophie	Wait, we did not yet formulate the rule…	
[14]	Adva	When there is an expression with both plus and minus …	As Adva is saying this, Sophie writes her words down on the worksheet.
[15]	Sophie	You drop the plus and you multiply the minus by the number that had the plus.	Continues writing while speaking.

Since the mathematical object *negative number* was introduced to the students just a few days earlier, there is clearly the need for meta-level learning, which requires trying to follow those in-the-know rather than just conducting one's own inventive explorations. We claim that for Sophie and Adva the situation is conducive to this type of learning because a learning-teaching agreement is also in place.

To begin with, although neither of the two protagonists seems to have, as yet, a satisfactory command over the new discourse, one of them is clearly the leader. Indeed, Sophie is somewhat more competent and sure of herself than her partner, and both girls seem to agree that her ideas should be given the lead. This fact expresses itself in Sophie's proactive behavior (see turns [3], [5], [7], [9], and [15], for example) which contrasts with Adva's tendency to just react to her partner's initiatives with challenges and requests for explanations ([4], [6], and [8]). Adva's doubts and questions notwithstanding, she eventually accepts Sophie's ideas, even if only tentatively ([10]).

As an aside, let us remark that Sophie overcomes the circularity of object construction by treating the new numbers as if they had properties of numbers with which she is already familiar. Thus, she reads the expression (+2)x(-5) the way she read, say 2x7, that is, as expressing whatever appears to the right of the little dot as a number that needs to be added to itself (see turn [5]). Most importantly, she treats the symbol -5 as referring to a single entity. For Adva, on the other hand, -5 appears to be a mere concatenation of a number and the minus ([8]).

The second requirement of a learning-teaching agreement appears to be fulfilled as well: Sophie is committed to her role of the leader ("teacher") and Adva acts as an ardent newcomer trying to get a genuine access to the new form of activity (learner). The latter student, although focused on her partner's ideas rather than offering her own, does not simply follow Sophie's lead in a mechanical manner. Her hesitation and the requests for explanations show her need for understanding the inner logic of the new discourse. Although the episode is too brief to allow for making definitive statements, it seems that Adva would not satisfy herself with purely ritualized participation and, in the longer run, will try to turn the discourse on negative numbers into a discourse-for-herself (at least, there is nothing in this present episode that would contradict this conjecture). Similarly, Sophie seems committed to her role as a leader: she answers Adva's questions as well as she can, and even if her answers would not always satisfy an expert interlocutor, this needs to be taken as a sign of her need for accountability and her commitment to mutual alignment.

Finally, the students seem to agree about the manner in which their learning should proceed. Above all, they do not delude themselves that the command over the new discourse and, in particular, the sense of its meaningfulness, usefulness, and coherence, may happen overnight. When Adva, still hesitant and not fully convinced in spite of Sophie's explanations decides to record the suggested solution ([10]), she comments, "We shall do it and then see." Although this phrase may refer to a future verdict by an authority, possibly the teacher, it may also indicate the student's readiness to suspend disbelief in the hope that clarity and a sense of coherence will emerge with discursive experience and practice. There is no reason to suspect that Sophie might disagree.

To sum up, in this episode the students are heading toward meta-level learning, and the situation seems favorable, in that the participants are in a full-fledged learning-teaching agreement. Since the necessary discursive change is time- and effort-consuming, one cannot but make conjectures about Sophie's and Adva's present and future learning. One thing we may say with a measure of confidence is that in our brief episode there are many signs that the required learning might, indeed, be taking place: the students have re-constructed for themselves the canonical rule of multiplication which, indeed, is to be learned, and they had some initial experience in applying this rule to several simple cases.

Example 2: When learning-teaching agreement is in place, but the leading discourse does not adhere to established rules

The same study provided us with an opportunity to see what may happen when students align themselves with a discourse that is not the one they were supposed to learn. The snippets that follow are taken from a whole-class discussion that followed work in pairs, of which episode 1 was an example. One of the students, Roi reports on the results of multiplying -2 by 5 obtained by him and his partner.

Episode 2: The class accepts Roi's rule for multiplying differently signed numbers

[1]	Roi	Minus ten	Answers the task «2× (-5) = ?» that was just written by the teacher on the blackboard.
[2]	Teacher	Because?	
[3]	Roi	We simply did two times five equals minus ten … because five is the bigger number, so ah .. like two times five is ten, but it's minus ten because we had minus five	
[4]	Teacher	I don't understand …	
[5]	Roi	We don't know how to explain that	
[6]	Teacher	Think about something, try to explain	
[7]	Leegal	I too think that it is minus ten … see, you do two times five and then you do two times minus, and this is minus ten	
[8]	Teacher	Why?	
[9]	Roi	Because five is bigger than two	

Slowly, Roi's non-standard proposal, its inexplicability notwithstanding, begins winning a general following. Only a few students object to what now seems to be a generally accepted rule of multiplication. Sophie is the only one to issue an open protest. Another student, Yasha, challenges Roi with a question, "And what if it is seven, not two [if the operation is 7x (-5) rather than 2x (-5)]?" Before Roi has a chance to react, several students volunteer their answers.

Episode 2 (continued): The class accepts Roi's rule for multiplying differently signed numbers

[10]	Vladis	If the seven is bigger, then it will be plus, and if the five is bigger, it's going to be minus.	
[11]	Guy	The explanation is that the bigger is the one which decides. Say, if seven is bigger, then it's plus.	
[12]	Teacher	You are only repeating what Roi said last time, but I need to know why you think it is so.	
[13]	Yoaz	Because this is what Roi is saying.	

Guided by questions *a*, *b*, and *c* formulated above, we can now analyze these rather surprising events in terms of the kind of learning that was supposed to take place and of the learning-teaching agreement between the participants.

As was stated already in Example 1, the discursive change the teacher was aiming at qualifies as meta-level, because new mathematical objects – negative numbers – had just been introduced and the students did not yet have a good sense of either the objects themselves or of meta-rules that guide endorsement or rejection of narratives about these objects.

In spite of the fact that none of the students seemed able to figure out the new meta-rules of endorsement (the teacher adamantly refrained from demonstrating her own discursive skills) – or perhaps just because of that – they had implicitly chosen Roi as their discursive leader. This choice might have more to do with Roi's being a "charismatic person" (as Roi described himself at a certain point) than with the nature of his arguments about the multiplication rule. This is, at least, what transpired from Yoaz's frank admission that the reason for his acceptance of the non-routine rule of multiplication was the sheer fact that "this [was] what Roi [was] saying" ([13]).

Not all those who agreed with Roi, though, did it in a thoughtless manner. Several students summoned the 'real-world' rule of "the bigger is the one who decides" ([11]; this argument surfaced recurrently throughout the lengthy whole class conversation). It thus seems justified to say that in the majority of cases, those who accepted Roi's leadership were genuinely interested in learning. The question whether Roi was equally interested in teaching is difficult to answer without additional data, but considering Roi's insistence and the explicitness with which he repeatedly described his ways of calculating throughout the conversation (see, for example, [3] and [9]), he was clearly quite keen to convert the children to his ideas. This might thus be a "missionary" type of teaching, but it was teaching nevertheless: Roi did want his classmates to follow in his discursive footsteps.

Finally, the students seemed to be in agreement about the ways in which the learning should take place. To say the least, there were no signs of uneasiness about either the fact that Roi clearly viewed his rule as acceptable even in the absence of any explanation ([5]), or about the nature and quality of the argument brought by some other students ([11]).

To sum up, in this present case, just as in the former one, a form of learning-teaching agreement does seem to exist. Moreover, we are witnessing meta-level learning of sorts: the children devise a rule for multiplying new kind of numbers, thus advancing the work of establishing these as-yet unfamiliar mathematical constructs. This learning, however, be it effective as it may, is not the kind of learning the teacher aimed for: the discourse which developed is not the one that would be recognized as proper by an expert discursant.

Example 3: When the potential discursive leader does not have a discourse to offer [13]

This time, we visit a sixth grade statistics lesson. In spite of their young age, the students are not altogether unacquainted with the topic: they have already implemented carefully designed statistical investigations in grades 4 and 5. At present, they are engaged in a lengthy task, the aim of which is to make them familiar with the idea of *representative sample* and *random sampling*. In the excerpt that follows, they are dealing with a small part of a multi-step assignment focusing on the notion of biased sample. They have been provided with a plastic bag containing shoe laces of differing lengths and were asked to calculate the mean (arithmetic average) of lace length in two subsequently drawn samples of ten laces. Later, they were to formulate a conjecture about the mean length of all the laces in the bag (the population mean). In the episode below, Dan and Anat are grappling with this last question [14].

Episode 3: Dan and Anat are trying to figure out the mean length of the lace in the bag.

[1]	Dan	OK, read number ten	Asks Anat to read question 10 in the worksheet.
[2]	Anat	«Based on your two samples, what is, in your opinion, the mean length of all the laces in the bag? Using the scale from 1 to 10, how certain are you of this assessment?»	Reads question 10 from the worksheet.
[3]	Dan	Aha, ok, what do you say about the mean of the whole bag? If we had twenty altogether… the means were 16 and 14 …… I think it is 18, something like that.	Refers to the two samples of ten laces that produced the means 16 and 14, respectively.
[4]	Anat	In my opinion, … I am certain of this … from 1 to 10 … seven, approximately.	Responds to the second part of the question about their confidence level in their answer.
[5]	Dan	Seven? That it is 18?	

[6]	Anat	No, I didn't say it was 18. It is how certain I am about what we did on the whole.	
[7]	Dan	No, no, … Remember? We had this question before? They asked what would be the mean of the whole bag.	Refers to a previous case in which they drew and assessed just one sample.
		So this is the same thing, but on the basis of two samples.	Points to the current question.
[8]	Anat		Looks intensely on the worksheet.
[9]	Dan	Aaaa…, I think that it is 18; the numbers we got are quite similar.	
[10]	Anat	You have to calculate, don't you think?	
[11]	Dan	See, first we got 14 in the first sample. Then we got 16 centimeter. So it cannot be, say, 28 or 30. One needs a closer number.	
[12]	Anat	But how do you know that this is exactly 18?	
[13]	Dan	I .. I don't know. I am not 100% sure of this	Shrugs his shoulders.
[14]	Anat	So perhaps we need to check this. One cannot just write a number.	
[15]	Dan	But if you had, say, to guess the mean of the whole bag…	
[16]	Anat	But they don't say here….	Silently reads the question.
		«In your opinion, what is the mean length of all the….»	Reads aloud.
[17]	Dan	This is the same as before	
[18]	Anat	The length!	Seems to have eventually understood what they were requested to do.
[19]	Dan	Yes	
[20]	Anat	Aha, the mean length of the laces in the bag.	
		Of all of them … But we didn't calculate them all.	Raises her head and looks at Dan.
[21]	Dan	Right. But judging from the two samples …	
		You remember the question …	Turns the worksheet and points to a question.
		It was exactly the same	
[22]	Anat	Aaa .., so 19, in my opinion	
[23]	Dan	… Okay, eighteen and a half?	
[24]	Anat and Dan		Giggling, the students turn to their respective worksheets and write down the agreed answer.

The discourse about statistical sampling is new to the students and so are, in particular, the notions of *sample* and *sampling variability*, which signal unfamiliar statistical objects. Some of the rules of the statistical game are new as well (in particular, statistical sampling), and their similarity to what the students were used to in mathematics classroom is deceptive. The learners are now invited to engage in the empirical work of measuring the laces rather than being given exact numerical data to manipulate in a formal way; and they are asked to "express their opinion" about certain magnitudes rather than actually calculate them. How the opinions can be shaped – whether they should be a result of mere guessing, of applying a formula to some numbers, or of making a well informed choice – is not immediately obvious to somebody who enters a statistics lesson directly from a mathematics classroom. It is therefore quite clear that the expected learning is going to happen first and foremost at the meta-level.

With regard to the learning-teaching agreement, this case appears similar to the one we saw in example 1. Like in the case of Sophie and Adva, one member of the pair, Dan, is visibly more proactive and shows more confidence, whereas the other student, Anat, is mainly reactive: she explores her partner's decisions, trying to understand why he is doing whatever he does. She may even be ready to follow in his footsteps, as long as Dan's answers make sense to her. Just as Adva challenged Sophie by presenting alternative possibilities, so does Anat challenge Dan by questioning his decisions, asking him why he chose one way of acting rather than another. Evidently inspired by the rules of mathematical discourse, Anat protests against what seems to her as Dan's arbitrary choice of the number 18, claiming that one should calculate numerical answers rather than just guess them - see [10] ("You have to calculate, don't you think?"), [12] ("But how do you know…?"), and [14] ("One cannot just write a number")[15].

At a closer look, however, the situation here may be somewhat different from what we saw in Example 1. This time, there is more symmetry between the students. Dan's proactive behavior notwithstanding, he is not necessarily considered as a leader. Indeed, his partner is much more reluctant to walk in his footsteps than Adva was to follow Sophie. Anat is clearly determined to make her own distinctive contribution when she states that the population mean might be 19, and not 18, as suggested by her partner. Above all, however, there seem to be no leading discourse here, not even in the most restricted of senses. When asked to raise conjecture about the population mean, none of the children seems to have a clear idea of how such a conjecture might be constructed. This stands in striking contrast with Sophie's (successful) effort to derive the new rule – that of multiplying negative number by positive – from the previously learned definition of multiplication. Evidently, Anat and Dan faced a similar question already in one of the former tasks and what Dan is now proposing is the obvious alternative to the principled inference: they will replicate what they did in this previous case ([7], [21]). Whereas the designers of the task expected the children to propose a number close to the mean of the two sample means, 14 and 16, the students opt for numbers that are slightly greater than 16. The rule that may guide this decision, "mean slowly grows with the number of items in the set" ([9], [11]), is never explicitly spelled out, let alone substantiated.

In the absence of well defined criteria with which to make and justify their choices and to resolve any possible difference of opinions, Anat and Dan arrive at a joint decision in a manner that has nothing to do with statistical considerations. Their awareness of this fact is probably the reason why they giggle while writing down their answer. The final act of averaging their proposals and ending up with the third number, 18.5 (see [23] and [24]) may be interpreted as a symbolic expression of the children's wish to align themselves with one another at any cost. In the absence of the leading discourse, however, this wish to agree can not qualify as a *learning-teaching* agreement. The incident leaves us also quite skeptical with respect to the effectiveness of Anat's and Dan's meta-level learning. The students seem to have acted in an ad hoc manner, without developing a rule, routine, or word use that would be applicable beyond this particular case.

Example 4: When there is well-defined leading discourse but the learning-teaching agreement is only partial

In the classroom episode that follows, taken from the Montreal Algebra Study[16], two 12-year old seven graders, Ari and Gur, are grappling with one of a long series of tasks supposed to usher them into algebraic thinking. On the worksheet in front of them, a linear function g(x) is introduced with the help of a partial table of values, and the students are required to find the value of g(6), which does not appear in this table. Using the given data, Ari has just found the slope (5) and the intercept (- 5) of g(x). The excerpt below begins when he writes down the formula of the function.

Episode 4: Ari and Gur calculate a value of a function given by a table.

[1]	Ari		Writes 5x+-5 on his worksheet.
[2]	Gur	What's that?	
[3]	Ari	It's the formula, so you can figure it out.	
[4]	Gur	Oh. How'd you get that formula?	
[5]	Ari	And you replace the x by 6.	Suggests how to solve the next task, i.e., find g(6).
[6]	Gur	Oh. Ok, I	

[7]	Ari	Look. Cause the, um the slope, is the zero. Ah, no, the intercept is the zero.	
[8]	Gur	Oh, yeah, yeah, yeah. So you got your	
[9]	Ari	And then you see how many is in between each, like from zero to what	«each»: Ari points to both columns, indicating that you have to check both; «from zero to what»: he points at the x column;
[10]	Gur	And the slope is, so the slope is 1.	The left counterpart of the right-column 0 is 1.
[11]	Ari	Hum? No, the slope, see you look at zero,	«zero»: Ari circles the zero n the x column on Gur's worksheet.
[12]	Gur	Oh that zero, ok. So the slope is minus 5	-5 is the g(x) value when x = 0 (g(0)=-5).
[13]	Ari	Yeah. And	
[14]	Gur	How are you supposed to get the other ones?	
[15]	Ari	You look how many times it's going down, like we did before. So it's going down by ones. So then it's easy. This is ah by fives. See, it's going down by ones, so you just look here	Ari first points to the x column («going down by ones»), then the g(x) column («by fives»), and again to g(x) column («look here»).
[16]	Gur	Oh. So it's 5	
[17]	Ari	Yeah. 5x plus	
[18]	Gur	Negative 5.	
[19]	Ari	Do you understand?	
[20]	Gur	Negative 5. Yeah, yeah, ok. So what is g 6?	
[21]	Ari	5 times 6 is 30, plus negative 5 is 25. So we did get it right.	
[22]	Gur	No, but it's - in this column there?	«this column»: Gur points to the x column.
[23]	Ari	Yeah	
[24]	Gur	Oh, then that makes sense. (writes) It's 30. What is g 10? … 40	
[25]	Ari	20, ah 40. No, 45	
[26]	Gur	No,	
[27]	Ari	45	
[28]	Gur	Because 20	
[29]	Ari	10 times 5 is 50, minus	
[30]	Gur	Well, 5 is 20, so 10 must have 40	Points to the two entries in the last row.
[31]	Ari	times 5	Circles the 10 in g(10) on Gur's sheet
[32]	Gur	Oh, we do that thing. Ok, just trying to find it.	
[33]	Ari	Yeah	
[34]	Gur	Cause I was thinking… cause 5 is 20,	Points again to the last row of the table.
[35]	Ari	It's 45. Yeah	
[36]	Gur	(mumbles) So it's 45.	

As before, let us try to answer the three questions about the nature of the expected learning, the quality of learning-teaching agreement, and the relation between the two. Since algebraic discourse at large and the notion of function in particular are new to the students, much meta-level learning has to occur before they gain a reasonable command over this new form of commognitive activity. Indeed, algebra has its own objects and its own rules of doing things, many of which are quite unlike anything the students have been accustomed to so far. Function, being their first tool for dealing with changing rather than constant magnitudes, is a particularly innovative kind of mathematical entity. Because of the previously mentioned circularity of the process of object construction, much effort must be invested in coming to grips with this unfamiliar type of mathematical "thing" (see e.g., Harel & Dubinsky, 1992).

Considering the meta-discursive nature of the expected change, it is not surprising that at least one of our present protagonists, Gur, is still groping in the dark. His partner's considerable proficiency in dealing with questions regarding function stands out against this contrasting background. In this situation, it is only natural for Ari to take the discursive lead and for Gur to assume the role of the learner. On the face of it, this is exactly what is happening. Gur seems to recognize Ari's discourse as the model to follow. In his attempt to understand what Ari is doing Gur persists in interrogating his partner ([2], [4], [14], [20], [22]), and although Ari's explanations do not seem truly effective (see Gur's response, "g(6) = 30" in [24], and his insistence that g(10) = 40 in [24], [26], and [30])), he eventually renounces his own solution for the sake of those proposed by his more competent partner ([36]).

If not for this obvious ineffectiveness of Gur's effort to make sense of Ari's solutions and of Ari's attempts to help, we might have concluded that the children are in a full agreement about the learning-teaching process. Gur's apparent failure to learn, however, compels us to give additional thought to the question of the quality of this agreement. Once we take a closer look at Ari and Gur's interaction, it becomes clear that the process of discursive rapprochement is hindered by both students' reluctance to assume the respective roles of student and teacher, but especially by Ari's unwillingness to take responsibility for Gur's learning. Although superficially polite, Ari does not really respond to Gur's questions - see, for example, how in [5] he continues the sentence he had begun in [3] in spite of Gur's interjected query [4]; even when he does respond, his answers show that he has not really listened to what Gur was saying (as evidenced by his confirmation in [13] of Gur's erroneous claim [12]); even when he inquires about Gur's understanding ([19]), he is not really interested in Gur's answer – if he were, he would have noticed that Gur's "Yeah, Yeah, ok" in [20] is immediately followed by a question that reveals complete lack of understanding of what Ari has been trying to explain); and finally, he ignores Gur's attempts to explain his own thinking – see, in particular the segment [28] – [34]. Except for Ari's implicit refusal to play the role of the teacher, the obvious ineffectiveness of Gur's learning can be explained by the boy's visible attempts to mask his role of a follower. Probably to conceal his dependence on Ari's guidance, he does not persist in his requests for explanations and claims understanding even when, in fact, he is unable to make any sense of what his partner has been saying (see e.g., [20]). He is also the one to initiate the transition to the next assignment, his (awkwardly camouflaged) inability to solve the former one notwithstanding ([24]).

To sum up, in spite of their implicit agreement about the discourse that should be given the lead, Ari and Gur's learning-teaching agreement is seriously impaired. In this situation, it is not surprising that there is almost no progress in Gur's discursive competence either in this brief classroom episode or, for that matter, in any other instance of Ari and Gur's collaboration kept in our records.

Coda: How to Interpret the Request for Autonomy in Learning?

It is now time to return to the questions asked in the beginning of this article: In participationist classroom, how can we interpret the request for the learner's autonomy? Which of the possible interpretations would be most beneficial for the student and which interpretations should be disqualified in advance? On the basis of the foregoing theoretical discussion and of empirical episodes analyzed above we now wish to claim that while answering these questions, we need to make a clear distinction between object-level and meta-level learning.

According to Yackel & Cobb (1996),

> … [S]tudents who are autonomous in mathematics are aware of, and draw on, their own intellectual capabilities when making mathematical decisions and judgments as they participate in these practices (Kamii, 1985). These students can be contrasted with those who are intellectually heteronomous and who rely on the pronouncements of an authority to know how to act appropriately. (p. 473)

Can mathematics students, indeed, restrict themselves to their own judgments? Is this call for refraining from appeals to authority realistic? In the case of object-level learning the answer to both these queries is the straightforward *yes*. When the students are already familiar with a given type of discourse and the goal of the learning is merely to explore its objects and to add to what is known about their properties, no special precautions are necessary in interpreting the calls for autonomy: The well-defined, familiar rules of the discourse should be sufficient, at least in principle, to let the student make independent, reasoned choices and to help her in resolving any possible dilemma. Close acquaintance with the inner logic of the discourse exempts the learner from the need to consult with others while making discursive decisions.

The situation changes when learning is supposed to happen at the meta-level. In this case, the exhortations to let the student "draw on their own intellectual capabilities" and not to "rely on the pronouncements of authority" do not go without saying. In particular, one must make it explicitly clear that they should not be interpreted as promoting Daedalus' principle of learners' unrestricted freedom. When issued in the context of meta-level learning, these two exhortations must thus be carefully explained and qualified. The first thing to stress in this context is that the two principles make no claims to exclusivity: student's own judgment is not expected to be the *sole* source of mathematical insights, and refraining from listening to those who count as authority is advisable only as long as there are clear, generally adopted criteria for discursive decisions. Claiming otherwise would contradict the basic participationist tenet that mathematics is historically established activity shaped by human choices rather than by a transcendental necessity. The view that students may, indeed, make a meta-level progress without an initial exposure to the discourse of experts could thus be sustained only by those who adopt the Platonic vision of mathematics and, in fact, negate the existence of meta-level learning as described in this article.

According to basic commognitive tenets, meta-level learning happens by scaffolded individualization, and for the scaffolding to be effective, there is the need for learning-teaching agreement between newcomers to the discourse and this discourse's expert oldtimers. Our examples have shown what may happen when one or more of the three components of learning-teaching agreement are missing or distorted, as is the case when, for example, the leading discourse is different from the one that is practiced by experienced, authoritative interlocutors (example 2); when there is no leading discourse and no generally accepted discursive authority (example 3); or when the leading discourse is in place but no expert support is available to the learner (example 4). In all these cases, the expected meta-level learning is unlikely to happen. When the students are left to themselves at the most crucial developmental crossroads, there is no reason to assume that they will turn in the same direction as the path-breakers of the past. At this point, all the options appear equally admissible, with none of them having any obvious advantage over the others. Left to their own inventiveness, the newcomers are likely to reach a dead end, as was the case in examples 2 and 3 (for the sake of proper disclosure we need to add that in both cases the following classroom discussion and the students' own further efforts helped the children to find their way out of the entanglement and their story did have happy ending, after all). In this situation, the act of turning to an expert, authoritative participant, often disparaged as a sign of students "thoughtlessness" is, in fact, the only alternative.

None of this, however, implies that in the case of meta-level learning there is no room for the learners' autonomy. To begin with, there is no contradiction between the seemingly incompatible scenarios of "drawing on one's own capabilities" and of "relying on authority," provided the word "relying" is not taken as tantamount to following Ariadne's string in a mechanical manner. Learner's autonomy is sustained as long as the exposure to expert forms of discourse is treated as the beginning rather than end of the learning process. The new discursive ways, as demonstrated by experts, will now become an object of exploration. The question, therefore, that one needs to ask herself while trying to assess learners' autonomy is not *whether* the students follow authoritative, expert discursants, but rather *how* they are doing this. Is this a thoughtless, ritualized following or an exercise in rationalization and in critical thinking? Or, to use Bakhtin's words, do they make a sincere effort to turn *authoritative discourse* into i*nternally persuasive* discourse? If the latter, the learner enjoys as much autonomy and has as much agency as when performing independent object-level explorations. In this way, she will eventually turn the discourse-for-others into a discourse-for-oneself.

Finally, let us remark that some important, and these days seemingly endangered human values are being cultivated in the process of learning that supports the autonomy of the learner, understood as explained above. If this autonomy is to be genuine and substantiable, much mutual respect must exist among participants. Student's sincere attempts to find reasons for expert choices is an expression of the newcomer's appreciation for oldtimers' thinking, and of her deep a priori confidence in the value of their time-honored discourse, painstakingly constructed throughout history.

NOTES

1. Indeed, Theseus did make it safely to freedom.

2. According to the legend, Daedalus instructed Icarus not to fly too close to the sun, lest the wax that kept his wings together melt. Forgetful of his father's warning and intoxicated with the feeling of freedom brought by the flying, Icarus rose all too high. The wax gave in and the boy fell into the sea.

3. The term *meta-rule* is used here to distinguish between the rules that govern the mathematical objects themselves, and the rules that regulate the activity of exploring these objects. The prefix meta- was added in this latter case to stress that these are rules of the discourse itself.

4. It can be shown that the process of construction, called objectification, involves *reification* - translation of the talk about processes into the talk about objects (substitution of nouns for verbs); and alienation – transition to impersonal, "agentless" form of talk (Sfard, in press).

5. Think, for example, about the statement, "multiplication makes bigger", which is fully endorsable in the discourse on natural numbers, but stops being so in the discourse on rational numbers.

6.The Kuhnian distinction between growth of knowledge in the period of *normal science* versus its development during *scientific revolution* (Kuhn, 1962) is the historical counterpart of the distinction between object-level and meta-level learning.

7. The term *discourse-for-oneself* is close to Vygotsky's idea of *speech-for-oneself*, introduced to denote a stage in the development of children's language (see e.g., Vygotsky, 1987, p. 71). These ideas also brings to mind the Bakhtinian distinction between *authoritative discourse*, a discourse that "binds us, quite independently of any power it might have to persuade us internally"; and *internally persuasive* discourse, one that is "tightly woven with 'one's own world" (Bakhtin, 1981, pp. 110-111).

8. The notion of *learning-teaching agreement* can be seen as a communicational counterpart and elaboration of Brousseau's idea of *didactic contract*, that is, of "the system of [students' and teachers'] reciprocal obligations" (Brousseau, 1997, p. 31). We do not claim that such agreement is sufficient for the success in overcoming the communicational gap – we only say it is crucially important for learning. This is a theoretical assertion, analytically derived from basic tenets of our approach. The empirical vignettes in the further part of this article seem to corroborate this claim.

9. As Vygotsky (1978) reminds us, a sociocultural vision of learning (and, in particular, his own notion *zone of proximal development*) must result in "reevaluation of the role of imitation in learning" (p. 87).

10. This study was conducted in collaboration with Sharon Avgil. The episode is the authors' translation from the Hebrew original. For the full report about this study see Sfard (2007).

11. Arrow model presents a signed number as a vector on the number line, the length of which equals the absolute value of the number and the direction of which corresponds to its sign (the vector corresponding to a negative number points left and the one corresponding to the positive points right). Magic cubes are entities which, when inserted into a liquid, increase (in case of the positive numbers) or decrease (for negative numbers) the temperature of this liquid by one degree.

12. This is a verbatim translation, but in English this expression could also be presented as "negative five." There is no equivalent of "negative five" in Hebrew. In our translation, we will alternate between the two forms for the sake of clarity. For example, in Hebrew, the word minus (which does appear exactly in this form) is not the only word for subtraction, as it is in English (the additional Hebrew word for this operation is *pachot*). In translating Adva's turn [2] (2 *pachot minus 5*) we used "negative five" rather than "minus five" simply to make clear that the minus here is supposed to signify the operation of subtraction. In fact, Adva's expression for -5 was no different than Sophie's in turn [1].

13. This example is taken from a three-year long design experiment in learning statistics led by Dani Ben-Zvi in collaboration with Naomi Apel and Einat Gil. The transcripts have been translated from Hebrew by the authors of this article. For more information on this longitudinal development and research project see Ben-Zvi (2006), and Ben-Zvi, Gil, & Apel (2007).

14. This will be later compared with the actual mean (the parameter), calculated on the basis of an exhaustive measurement. The intention is to make the students aware that because of the differing lengths of the laces, and because the longer among them are more likely to be drawn, the mean of the samples was somewhat higher than the true population mean.

15. Another possibility needs to be considered that up to the utterances 18 and 20, Anat might have not had the proper understanding of the assignment and, more specifically, might have been unaware that the task was to assess (as opposed to calculate) the population mean, whereas not all the values in this greater set were known. If this is true, then Anat's surprise at what Dan was doing is even more understandable, and the existence of the agreement on the leading discourse, discussed in the next paragraph, is even more questionable.

16. This study, co-directed by Carolyn Kieran and Anna Sfard, has been implemented in 1992-1994 in one of the Montreal middle schools, situated in an affluent area. The aim of the 30 session long teaching sequence produced for the sake of the study was to introduce the students to algebra while investigating their ways of developing algebraic discourse and testing certain hypotheses about possible ways of spurring this development. The present episode is taken from the 21 st meeting. More information on the study, as well as another outlook at the present episode, may be found in Kieran & Sfard (1999), Sfard & Kieran (2001) and Sfard (in press).

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