# Chapter 6

# **REASONING ABOUT DATA ANALYSIS**

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# OVERVIEW

The purpose of this chapter is to describe and analyze the ways in which middle school students begin to reason about data and come to understand exploratory data analysis (EDA). The process of developing reasoning about data while learning skills, procedures, and concepts is described. In addition, the students are observed as they begin to adopt and exercise some of the habits and points of view that are associated with statistical thinking. The first case study focuses on the development of a global view of data and data representations. The second case study concentrates on design of a meaningful EDA learning environment that promotes statistical reasoning about data analysis. In light of the analysis, a description of what it may mean to learn to reason about data analysis is proposed and educational and curricular implications are drawn.

# THE NATURE OF EXPLORATORY DATA ANALYSIS

Exploratory data analysis (EDA), developed by Tukey (1977), is the discipline of organizing, describing, representing, and analyzing data, with a heavy reliance on visual displays and, in many cases, technology. The goal of EDA is to make sense of data, analogous to an *explorer of unknown lands* (Cobb & Moore, 1997). The original ideas of EDA have since been expanded by Mosteller and Tukey (1977) and Velleman and Hoaglin (1981); they have become the accepted way of approaching the analysis of data (Biehler, 1990; Moore, 1990, 1992).

According to Graham (1987), and Kader and Perry (1994), data analysis is viewed as a four-stage process: (a) pose a question and formulate a hypothesis, (b) collect data, (c) analyze data, and (d) interpret the results and communicate conclusions. In reality however, statisticians do not proceed linearly in this process, but rather iteratively, moving forward and backward, considering and selecting possible paths (Konold & Higgins, 2003). Thus, EDA is more complex than the four-stage process: "data analysis is like a give-and-take conversation between the

hunches researchers have about some phenomenon and what the data have to say about those hunches. What researchers find in the data changes their initial understanding, which changes how they look at the data, which changes their understanding" (Konold & Higgins, 2003, p. 197).

EDA employs a variety of techniques, mostly graphical in nature, to maximize insight into a data set. Exploring a data set includes examining shape, center, and spread; and investigating various graphs to see if they reveal clusters of data points, gaps, or outliers. In this way, an attempt is made to uncover underlying structure and patterns, test underlying assumptions, and develop parsimonious models. Many EDA graphical techniques are quite simple, such as stem-and-leaf plots and box plots. Computers support EDA by making it possible to quickly manipulate and display data in numerous ways, using statistical software packages such as Data Desk (Velleman, 2003), Fathom (Finzer, 2003), and Tabletop (TERC, 2002).

However, the focus of EDA is not on a set of techniques, but on making sense of data, how we dissect a data set; what we look for; how we look; and how we interpret. EDA postpones the classical "statistical inference" assumptions about what kind of model the data follow with the more direct approach of "let the numbers speak for themselves" (Moore, 2000, p. 1), that is, allowing the data itself to reveal its underlying structure and model.

This complete and complex picture of data analysis should be reflected in the teaching of EDA and in the research on students' statistical reasoning. Simplistic views can lead to the use of recipe approaches to data analysis instruction and to research that does not go beyond the surface understanding of statistical techniques.

#### EDA in School Curriculum

EDA provides a pedagogical opportunity for open-ended data exploration by students, aided by educational technology. Allowing students to explore data is aligned with current educational paradigms, such as, teaching and learning for understanding (Perkins & Unger, 1999), inquiry-based learning (Yerushalmy, Chazan, & Gordon, 1990), and project-based learning (Evensen & Hmelo, 2000). However, the complexity of EDA raises numerous instructional challenges, for example, how to teach methods in a new and changing field, how to compensate for the lack of teachers' prior experience with statistics, and how to put together an effective K–12 curriculum in statistics that incorporates EDA.

Elements of EDA have been integrated into the school mathematics curriculum in several countries, such as Australia (Australian Education Council, 1991, 1994), England (Department for Education and Employment, 1999), New Zealand (Ministry of Education, 1992), and the United States (National Council of Teachers of Mathematics, 1989, 2000). In recently developed curricula—for example, *Chance and Data* (Lovitt & Lowe, 1993), *The Connected Mathematics Project* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1996), *Data: Kids, Cats, and Ads* (Rubin & Mokros, 1998), *Data Handling* (Greer, Yamin-Ali, Boyd, Boyle, & Fitzpatrick, 1995), *Data Visualization* (de Lange & Verhage, 1992), *Exploring Statistics* (Bereska, Bolster, Bolster, & Scheaffer, 1998, 1999), *The Quantitative Literacy*  *Series* (e.g., Barbella, Kepner, & Schaeffer, 1994), and *Used Numbers* (e.g., Friel, Mokros, & Russel, 1992)—there is growing emphasis on developing students' statistical reasoning about data analysis; on graphical approaches; on students gathering their own data and intelligently carrying out investigations; on the use of educational software, simulations, and Internet; on a cross-curricular approach; and on the exploration of misuses and distortions as points of departure for study.

#### Research on Reasoning about Data Analysis

Research on reasoning about data analysis is beginning to emerge as a unique area of inquiry. In a teaching experiment conducted with lower secondary school students by Biehler & Steinbring (1991), data analysis was introduced as "detective" work. Teachers gradually provided students with a data "tool kit" consisting of tasks, concepts, and graphical representations. The researchers concluded that all students succeeded in acquiring the beginning tools of EDA, and that both the teaching and the learning became more difficult as the process became more open. There appears to be a tension between directive and nondirective teaching methods in this study. A study by de Lange, Burrill, & Romberg (1993) reveals the crucial need for professional development of teachers in the teaching of EDA in the light of the difficulties teachers may find in changing their teaching strategy from expository authority to guide. It is also a challenge for curriculum developers to consider these pedagogical issues when creating innovative EDA materials. Recent experimental studies in teaching EDA around key concepts (distribution, covariation) in middle school classes have been conducted by Cobb (cf., 1999) with an emphasis on sociocultural perspectives of teaching and learning.

Ben-Zvi and Friedlander (1997b) described some of the characteristic reasoning processes observed in students' handling of data representations in four patterns: (a) *uncritical thinking*, in which the technological power and statistical methods are used randomly or uncritically rather than "targeted"; (b) *meaningful use of a representation*, in which students use an appropriate graphical representation or measure in order to answer their research questions and interpret their findings; (c) *meaningful handling of multiple representations*, in which students are involved in an ongoing search for meaning and interpretation to achieve sensible results as well as in monitoring their processes; and (d) *creative thinking*, in which students decide that an uncommon representation or method would best express their thoughts, and they manage to produce an innovative graphical representation, or self-invented measure, or method of analysis.

# THE CURRENT STUDY

#### **Theoretical Perspectives**

Research on mathematical cognition in the last decades seems to converge on some important findings about learning, understanding, and becoming competent in mathematics. Stated in general terms, research indicates that becoming competent in a complex subject matter domain, such as mathematics or statistics, "may be as much a matter of acquiring the habits and dispositions of interpretation and sense making as of acquiring any particular set of skills, strategies, or knowledge" (Resnick, 1988, p. 58). This involves both cognitive development and "socialization processes" into the culture and values of "doing mathematics" (*enculturation*). Many researchers have been working on the design of teaching in order to "bring the practice of knowing mathematics in school closer to what it means to know mathematics within the discipline" (Lampert, 1990, p. 29). This chapter is intended as a contribution to the understanding of these processes in the area of EDA.

# Enculturation Processes in Statistics Education

A core idea used in this study is that of *enculturation*. Recent learning theories in mathematics education (cf., Schoenfeld, 1992; Resnick, 1988) include the process of enculturation. Briefly stated, this process refers to entering a community or a practice and picking up their points of view. The beginning student learns to participate in a certain cognitive and cultural practice, where the teacher has the important role of a mentor and mediator, or the *enculturator*. This is especially the case with regard to statistical thinking, with its own values and belief systems and its habits of questioning, representing, concluding, and communicating. Thus, for *statistical enculturation* to occur, specific thinking tools are to be developed alongside collaborative and communicative processes taking place in the classroom.

# Statistical Thinking

Bringing the practice of knowing statistics at school closer to what it means to know statistics within the discipline requires a description of the latter. Based on indepth interviews with practicing statisticians and statistics students, Wild and Pfannkuch (1999, and Chapter 2) provide a comprehensive description of the processes involved in statistical thinking, from problem formulation to conclusions. They suggest that a statistician operates (sometimes simultaneously) along four dimensions: investigative cycles, types of thinking, interrogative cycles, and dispositions.

Based on these perspectives, the following research questions were used to structure the case studies and the analysis of data collected:

- How do junior high school students begin to reason about data and make sense of the EDA perspective in the context of open-ended problem-solving situations, supported by computerized tools?
- How do aspects of the learning environment promote students' statistical reasoning about data analysis?

# METHOD

This study employs a qualitative analysis method, to examine seventh-grade students' statistical reasoning about data in the context of two classroom investigations. Descriptions of the setting, curriculum, and technology are followed by a profile of the students, and then by methods of data collection and analysis.

## The Setting

The study took place in three seventh-grade classes (13-year-old girls and boys) in a progressive experimental school in Tel-Aviv. The classes were taught by skillful and experienced teachers, who were aware of the spirit and goals of the curriculum (described briefly later). They were part of the CompuMath curriculum development and research team, which included several mathematics and statistics educators and researchers from the Weizmann Institute of Science, Israel. The CompuMath Project is a large and comprehensive mathematics curriculum for grades 7–9 (Hershkowitz, Dreyfus, Ben-Zvi, Friedlander, Hadas, Resnick, Tabach, & Schwarz, 2002), which is characterized by the teaching and learning of mathematics using open-ended problem situations to be investigated by peer collaboration and classroom discussions using computerized environments.

The *Statistics Curriculum* (SC)—the data component of the CompuMath Project—was developed to introduce junior high school students (grade 7, age 13) to statistical reasoning and the "art and culture" of EDA (described in more detail in Ben-Zvi & Friedlander, 1997b). The design of the curriculum was based on the creation of small scenarios in which students can experience some of the processes involved in the experts' practice of data-based enquiry. The SC was implemented in schools and in teacher courses, and subsequently revised in several curriculum development cycles.

The SC was designed on the basis of the theoretical perspectives on learning and the expert view of statistical thinking just described. It stresses: (a) student's active participation in organization, description, interpretation, representation, and analysis of data situations (on topics close to the students' world such as sport records, lengths of people's names in different countries, labor conflicts, car brands), with a considerable use of visual displays as analytical tools (in the spirit of Garfield, 1995, and Shaughnessy, Garfield, & Greer, 1996); and (b) incorporation of technological tools for simple use of various data representations and transformations of them (as described in Biehler, 1993, 1997; Ben-Zvi, 2000). The scope of the curriculum is 30

periods spread over 2-1/2 months, and it includes student book (Ben-Zvi & Friedlander, 1997a) and teacher guide (Ben-Zvi & Ozruso, 2001).

# Technology

During the experimental implementation of the curriculum a spreadsheet package (Excel) was used. Although Excel is not the ideal tool for data analysis (Ben-Zvi, 2000), the main reasons for choosing this software were:

- Spreadsheets provide direct access that allows students to view and explore data in different forms, investigate different models that may fit the data, manipulate a line to fit a scatter plot, etc.
- Spreadsheets are flexible and dynamic, allowing students to experiment with and alter representations of data. For instance, they may change, delete or add data entries in a table and consider the graphical effect of the change or manipulate directly data points on the graph and observe the effects on a line of fit. Spreadsheets are adaptable by providing control over the content and style of the output.
- Spreadsheets are common, familiar, and recognized as a fundamental part of computer literacy (Hunt, 1995). They are used in many areas of everyday life, as well as in other domains of mathematics curricula, and are available in many school computer labs. Hence, learning statistics with a spreadsheet helps to reinforce the idea that this is something connected to the real world.

# Participants

This study focuses mainly on two students—A and D (in the first case), and on A and D and four of their peers (in the second case). A and D were above–average ability students, very verbal, experienced in working collaboratively in computerassisted environments, and willing to share their thoughts, attitudes, doubts, and difficulties. They agreed to participate in this study, which took place within their regular classroom periods and included being videotaped and interviewed (after class) as well as furnishing their notebooks for analysis.

When they started to learn this curriculum, A and D had limited in-school statistical experience. However, they had some informal ideas and positive dispositions toward statistics, mostly through exposure to statistics jargon in the media. In primary school, they had learned only about the mean and the uses of some diagrams. Prior to, and in parallel with, the learning of the SC they studied beginning algebra based on the use of spreadsheets to generalize numerical linear patterns (Resnick & Tabach, 1999).

The students appeared to engage seriously with the curriculum, trying to understand and reach agreement on each task. They were quite independent in their work, and called the teacher only when technical or conceptual issues impeded their progress. The fact that they were videotaped did not intimidate them. On the contrary, they were pleased to speak out loud; address the camera explaining their actions, intentions, and misunderstandings; and share what they believed were their successes.

# Data Collection

To study the effects of the new curriculum, student behavior was analyzed using video recordings, classroom observations, interviews, and the assessment of students' notebooks and research projects. The two students—A and D—were videotaped at almost all stages (20 hours of tapes), and their notebooks were also collected.

# Analysis

The analysis of the videotapes was based on interpretive microanalysis (see, for example, Meira, 1991, pp. 62–63): a qualitative detailed analysis of the protocols, taking into account verbal, gestural and symbolic actions within the situations in which they occurred. The goal of such an analysis is to infer and trace the development of cognitive structures and the sociocultural processes of understanding and learning.

Two stages were used to validate the analysis, one within the CompuMath researchers' team and one with four researchers from the Weizmann Institute of Science, who had no involvement with the data or the SC (triangulation in the sense of Schoenfeld, 1994). In both stages the researchers discussed, presented, and advanced and/or rejected hypotheses, interpretations, and inferences about the students' cognitive structures. Advancing or rejecting an interpretation required: (a) providing as many pieces of evidence as possible (including past and/or future episodes, and all sources of data as described earlier); and (b) attempts to produce equally strong alternative interpretations based on the available evidence. In most cases the two analyses were in full agreement, and points of doubt or rejection were refuted or resolved by iterative analysis of the data.

# Case Study 1: Constructing Global Views of Data

The first case study concentrates on the growth and change of the students' conceptions as they entered and learned the culture of EDA and started to develop their reasoning about data and data representations. This study focused on the shift between local observations and global observations. In EDA, *local understanding* of data involves focusing on an individual value (or a few of them) within a group of data (a particular entry in a table of data, a single point in a graph). *Global understanding* refers to the ability to search for, recognize, describe, and explain general patterns in a set of data (change over time, trends) by naked-eye observation of distributions and/or by means of statistical parameters or techniques. Looking globally at a graph as a way to discern patterns and generalities is fundamental to

statistics, and it includes the production of explanations, comparisons, and predictions based on the variability in the data. By attending to where a collection of values is centered, how those values are distributed or how they change over time, statistics deals with features not inherent to individual elements but to the aggregate that they comprise.

Learning to look globally at data can be a complex process. Studies in mathematics education show that students with a sound local understanding of certain mathematical concepts struggle to develop global views (cf., Monk, 1988; Bakker, Chapter 7). Konold, Pollatsek, and Well (1997) observed that high school students—after a yearlong statistics course—still had a tendency to focus on properties of individual cases rather than on propensities of data sets.

The interplay between local and global views of data is reflected in the tools statistics experts use. Among such tools, which support data-based arguments, explanations, and (possibly) forecasts, are *time plots*, which highlight data features such as trends and outliers, center, rates of change, fluctuations, cycles, and gaps (Moore & McCabe, 1993). For the purpose of reflection (or even dishonest manipulation), trends can be highlighted or obscured by changing the scales. For example, in Cartesian-like graphs the vertical axis can be "stretched," so that the graph conveys the visual impression of a steep slope for segments connecting consecutive points, giving a visual impression that the rate of change is large. Experts propose standards in order to avoid such visual distortions (cf., Cleveland, 1994, pp. 66–67).

# The Task

In the first activity of the SC, the *Men's 100 Meters Olympic Race*, students were asked to examine real data about the winning times in the men's 100 meters during the modern Olympic Games. Working in pairs, assisted by the spreadsheet, they were expected to analyze the data in order to find trends and interesting phenomena. This covariation problem concerned tables and graphical representations (time plots) and formulating verbal statements as well as translating among those representations. In the second part of this activity, a problem is presented to students in the following way:

Two sports journalists argue about the record times in the 100 meters. One of them claims that there seems to be no limit to human ability to improve the record. The other argues that sometime there will be a record, which will never be broken. To support their positions, both journalists use graphs.

One task of this investigation asks students to design a representation, using a computer, to support different statements, such as: (a) The times recorded in the Olympic 100 meters improved considerably; and (b) Throughout the years, the changes in the Olympic times for the 100 meters were insignificant.

# Analysis: Toward an Expert Reasoning

Students started their introduction to EDA by learning to make sense of general questions normally asked in data exploration. They often offered irrelevant answers, revealed an implicit sense of discomfort with these answers, asked for help, and used the teacher's feedback to try other answers. They worked on EDA tasks with partial understanding of the overall goal. By confronting the same issues with different sets of data and in different investigational contexts, they overcame some of their difficulties. The teacher's role included reinforcing the legitimacy of an observation as being of the right "kind" despite not being fully correct, or simply refocusing attention on the question. These initial steps in an unknown field are regarded as an aspect of the enculturation process (e.g., Schoenfeld, 1992; Resnick, 1988).

At the beginning stage, students also struggled with how to read and make sense of local (pointwise) information in tables and in graphs. This stage involved learning to see each row in a table (Table 1) with all its details as one whole case out of the many shown, and focusing their attention on the entries that were important for the curricular goal of this activity: the record time, and the year it occurred. This view of each single row, with its two most relevant pieces of information, was reinforced afterward when students displayed the data in a time plot (Figure 1), since the graph (as opposed to the table) displays just these two variables. Also, this understanding of pointwise information served later on as the basis for developing a global view, as an answer to "how do records change over time?"

Year	City	Athlete's name	Country	Time (sec.)
1896	Athens	Thomas Burke	USA	12.0
1900	Paris	Francis Jarvis	USA	10.8
1904	St. Louis	Archie Hahn	USA	11.0
1908	London	Reginald Walker	South Africa	10.8
1912	Stockholm	Ralph Craig	USA	10.8
1920	Antwerp	Charles Paddock	USA	10.8
1924	Paris	Harold Abrahams	UK	10.6

Table 1. Part of the table of the men's 100 meters winning times in the 23 Olympiads from 1896 to 1996



Figure 1. Time plot showing winning times for men's 100 meters.

Instead of looking at the graph as a way to discern patterns in the data, students' response focused first on the nature and language of the graph as a representation—how it displays discrete data, rather than as a tool to display a generality, a trend. When invited to use the line connecting the dots in the dot plot (Figure 1) as an artifact to support a global view, they rejected it because it lacked any meaning in light of the pointwise view they had just learned, and with which they felt comfortable.

When A and D were asked to describe what they learned from the 100 meters table (Table 1), they observed that "There isn't anything constant here." After the teacher reinforced the legitimacy of their observation, they explained more clearly what they meant by constancy in the following dialogue (the dialogues are translated from Hebrew, therefore they may not sound as authentic as in the original):

- *D* Let's answer the first question: "What do you learn from this table?"
- A There are no constant differences between ...
- *D* We learn from this table that there are no constant differences between the record times of ... [looking for words]
- A The results of ...
- *D* The record times of the runners in ...
- A There are no constant differences between the runners in the different Olympiads ...

The students' attention focused on differences between adjacent pairs of data entries, and they noticed that these differences are not constant. These comparisons presumably stemmed from their previous knowledge and experiences with a spreadsheet in algebra toward finding a formula. In other words, one of the factors that moved them forward toward observation of patterns was their application of previous knowledge. Thus, the general pattern the students observed and were able to express was that the differences were not constant. Maybe they implicitly began to sense that the nature of these data in this new area of EDA, as opposed to algebra, is disorganized, and it is not possible to capture it in a single deterministic formula.

After the two students had analyzed the 100 meters data for a while, they worked on the next question: to formulate a preliminary hypothesis regarding the trends in the data. They seemed to be embarrassed by their ignorance—not knowing what trends mean, and asked for the teacher's help.

- *A* What are trends? What does it mean?
- *T* What is a trend? A trend is ... What's the meaning of the word trend?
- *A* Ah ... Yes, among other things, and what is the meaning in the question.
- T O.K. Let's see: We are supposed to look at what?
- *D* At the table.
- T At the table. More specifically—at what?
- A At the records.
- *T* At the records. O.K. And now, we are asked about what we see: Does it decrease all the time?
- A&D No.
- *T* No. Does it increase all the time?
- A&D No.
- *T* No. So, what does it do after all?
- D It changes.
- T It changes. Correct.
- *A* It generally changes from Olympiad to Olympiad. Generally, not always.
- *T* Sometimes it doesn't change at all. Very nice! Still, it usually changes. And, is there an overall direction?
- D No!
- *T* No overall direction?
- A There is no overall declining direction, namely, improvement of records. But, sometimes there is deterioration ...
- *T* Hold on. The overall direction is? Trend and direction are the same.
- A&D Increase!
- T The general trend is ...
- D Improvement in records.
- *T* What is "improvement in records"?
- *A* Decline in running times.
- *T* Yes. Decline in running times. O.K. ... But ...
- *A* Sometimes there are bumps, sort of steps ...
- T ... But, this means that although we have deviations from the overall direction here and there, still the overall direction is this ... Fine, write it down.

The students were unfamiliar with the term *trends*, and they were vague about the question's purpose and formulation. In response, the teacher gradually tried to nudge the students' reasoning toward global views of the data. Once they understood the intention of the question, the students—who viewed the irregularity

as the most salient phenomenon in the data—were somehow bound by the saliency of local values: They remained attached to local retrogressions, which they could not overlook in favor of a general sense of direction/trend.

The teacher, who did not provide a direct answer, tried to help them in many ways. First, she devolved the question (in the sense of Brousseau, 1997, pp. 33–35 and 229–235), and when this did not work, she rephrased the question in order to refocus it: "We are supposed to look at what?" and "more specifically at what?" She then hints via direct questions: "Does it increase all the time?" and "So, what does it do after all?" In addition, she appropriated (in the sense of Moschkovich, 1989) the students' answers to push the conversation forward by using their words and answers, for example: "It changes. Correct"; "increase"; "decrease." At other times she subtly transformed their language, such as changing *bumps* to *deviations;* or by providing alternative language to rephrase the original question to: "Is there an overall direction?"

After the interaction just presented, A and D wrote in their notebooks the following hypothesis: "The overall direction is increase in the records, yet there were occasionally lower (slower) results, than the ones achieved in previous Olympiads." At this stage, it seems that they understood (at least partially) the meaning of *trend*, but still stressed (less prominently than before) those local features that did not fit the pattern.

In the second part of the activity, the students were asked to delete an "outlying" point (the record of 12 sec. in the first Olympiad, 1896) from the graph (Figure 1) and describe the effect on its shape. The purpose of the curriculum was to lead students to learn how to transform the graph in order to highlight trends. It was found that by focusing on an exceptional point and the effect of its deletion directed students' attention to a general view of the graph. This finding seems consistent with Ainley (1995), who also describes how an outlier supported students' construction of global meanings for graphs.

The following transcript describes the students' comments on the effect of changing the vertical scales of the original 100 meters graph from 0-12 (Figure 2) to 0-40 (Figure 3) as requested in the second part of the activity.

- A Now, the change is that the whole graph stayed the same in shape, but it went down.
- *D* The same in shape, but much, much lower, because the column [the y-axis] went up higher. Did you understand that? [D uses both hands to signal the down and up movements of the graph and the y-axis respectively.]
- *A* Because now the 12, which is the worst record, is lower. It used to be once the highest. Therefore, the graph started from very high. But now, it [the graph] is already very low.



*Figure 2*. The original 100 meters graph. *Figure 3*. The 100 meters graph after the change of the y-scales.

The change of scales also focused the students' attention on the graph as a whole. They talked about the change in the overall relative position of the graph, whereas they perceived the shape itself as "the same." Their description included global features of the graph ("The whole graph ... went down"), attempts to make sense of the change via the y-axis ("Because the column went up higher"), and references to an individual salient point ("Because now the 12, which is the worst record, is lower"). Student A wrote the following synthesis in his notebook: "The graph remained the same in its shape, but moved downward, because before, 12—the worst record—was the highest number on the y-axis, but now it is lower."

However, the purpose of the rescaling was to enable the students to visualize the graph as a whole in a different sense. In order to take sides in the journalists' debate, the transformation was aimed at visually supporting the position that there are no significant changes in the records. Although the students' focus was global, for them the perceptually salient effect of the rescaling was on relative "location" of the whole graph rather than on its trend.

When *A* and *D* were asked to design a graph to support the (opposite) statement: "Over the years, the times recorded in the Olympic 100 meters improved considerably," they did not understand the task and requested the teacher's help:

- *T* [Referring to the 0–40 graph displayed on the computer screen—see Figure 3.] How did you flatten the graph?
- A [Visibly surprised.] How did we flatten it?
- T Yes, you certainly notice that you have flattened it, don't you?
- *D* No. The graph was like that before. It was only higher up [on the screen].

The teacher and the students seemed to be at cross purposes. The teacher assumed that the students had made sense of the task in the way she expected, and that they understood the global visual effect of the scaling on the graph's shape. When she asked, "How did you flatten the graph?" she was reacting to what she thought was their difficulty: how to perform a scale change in order to support the claim. Thus, her hint consisted of reminding them of what they had already done (scale change). However, the students neither understood her jargon ("flatten the

graph") nor regarded what they had done as changing the graph's shape ("The graph was like that before"). Although this intervention is an interesting case of miscommunication, it apparently had a catalytic effect, as reflected in the dialogue that took place immediately afterward—after the teacher realized what might have been their problem.

- *T* How would you show that there were very very big improvements?
- A [Referring to the 0–40 graph; see Figure 3.] We need to decrease it [the maximum value of the y-axis]. The opposite of ... [what we have previously done].
- *D* No. To increase it [to raise the highest graph point, i.e., 12 sec.].
- *A* The graph will go further down.
- *D* No. It will go further up.
- *A* No. It will go further down.
- D What you mean by increasing it, I mean—decreasing.
- A Ahhh ... Well, to decrease it ... O.K., That's what I meant. Good, I understand.
- *D* As a matter of fact, we make the graph shape look different, although it is actually the same graph. It will look as if it supports a specific claim.

When the teacher rephrased her comment ("How would you show that there were very very big improvements?") the students started to make sense of her remarks, although they were still attached to the up-down movement of the whole graph. Student D began to discern that a change of scale might change the perceptual impressions one may get from the graph. The teacher's first intervention ("How did you flatten the graph?"), although intended to help the students make sense of the task, can be considered unfortunate. She did not grasp the nature of their question, misjudged their position, and tried to help by reminding them of their previous actions on scale changing. The students seemed comfortable with scale changing, but their problem was that they viewed this tool as doing something different from what the curriculum intended.

The miscommunication itself, and the teacher's attempt to extricate herself from it, contributed to their progress. At first, A and D were surprised by her description of what they had done as *flattening* the graph. Then, they "appropriated" the teacher's point of view (in the sense of Moschkovich, 1989) and started directing their attention to the shape of the graph rather than to its relative position on the screen. They started to focus on scaling and rescaling in order to achieve the "most convincing" design. Briefly stated, they transferred and elaborated, in iterative steps, ideas of changing scales from one axis to the other until they finally arrived at a satisfying graph (Figure 4) with no further intervention from the teacher. (See Ben-Zvi, 1999, for a detailed description of this rescaling process.) Students A and D flexibly and interchangeably relied on pointwise observations and global considerations (both in the table and in the graph) in order to fix the optimal intervals on the axes so that the figure would look as they wished.



*Figure 4*. Graph designed to support the statement that the 100 meters times improved considerably.

In summary, at the beginning of this episode the students interpreted the effect of changing scales as a movement of the graph downward rather than as an effect on its shape. Following the teacher's intervention, they started to consider how scaling of both axes affects the shape of the graph. Moreover, they were able to develop manipulations for these changes to occur in order to achieve the desired shape. In the process, they began to move between local and global views of the data in two representations.

It is interesting to notice the students' persistent invocation of "differences" between values ("This way we actually achieved a result that appears as if there are enormous differences"). However, their focus here is on the way these differences are "blown up" by the scaling effect, rather than on them not being constant, as was the case earlier when differences were invoked. The importance of their prior knowledge appears to have been adapted to a new use and for a new purpose. The differences, which were used to drive the way the students made sense of patterns in the data, were being successfully used here as a powerful tool to evaluate their success in designing a graph to visually support a certain claim about a trend in the data.

# Case Study 2: Students Taking a Stand

The second case study focused on the role of the SC learning environment in supporting students' reasoning about data analysis. The students in this study were observed as they engaged in *taking a stand* in a debate on the basis of data analysis. The purpose of the analysis was to advance the understanding of (a) how students learn in such an environment, and (b) how can we be more aware of student

reasoning, in order to design "better" tasks. Better tasks are situations in which students engage seriously, work and reflect, and advance their statistical reasoning about data.

One SC activity was the Work dispute in a printing company. In this activity, the workers are in dispute with the management, which has agreed to an increase in the total salary amount by 10 percent. How this amount of money is to be divided among the employees is a problem-and thereby hangs the dispute. The students were given the salary list of the 100 employees, along with an instruction booklet to guide them in their work. They also received information about the national average and minimum salaries, Internet sites to look for data on salaries, and newspaper articles about work disputes and strikes. In the first part of the activity, students were required to take sides in the dispute and to clarify their arguments. Then, using the computer, they described the distribution of salaries and used statistical measures (e.g. median, mean, mode, and range) to support their position in the dispute. The students learned the effects of grouping data and the different uses of statistical measures in arguing their case. In the third part, the students suggested alterations to the salary structure without exceeding the 10 percent limit. They produced their proposal to solve the dispute, and designed representations to support their position and refute opposing arguments. Finally the class met for a general debate and voted for the winning proposal. The time spent on the full activity was about seven class periods, or a total of six hours.

This task context was familiar to students since it provided interesting, realistic, and meaningful data. The data were altered so that they were more manageable and provided points of departure for addressing some key statistical concepts. For example, the various central tendency measures were different, allowing students to choose a representative measure to argue their case. It was arranged that the mean salary (5000 IS) was above the real national averages (4350 IS—all employees, 4500 IS—printers only).

Students were expected to clarify their thoughts, learn to listen to each other, and try to make sense of each other's ideas. But, most importantly students were asked to *take sides* in the conflict situation. Their actions (e.g. handling data, choosing statistics, creating displays, and arguing) were all motivated, guided, and targeted by the stand they chose. However, their actions sometimes caused them to change their original stand.

The following transcript from a video recording of one of the experimental classes illustrates the use of concepts, arguments, and statitical reasoning that the task promoted. It is based on a group of students who chose to take the side of the workers. After clarifying their arguments, they described the distribution of the current salaries, guided by their position in the dispute. The student pairs prepared various suggested alterations to the salary structure to favor workers (as opposed to management), and then held a series of meetings with fellow student pairs (about 10 students in all), in which they discussed proposals, designed graphical representations to support their position, and prepared themselves for the general debate. This transcript is taken from the second "workers' meeting." It includes the students A and D from the previous case study along with four other students (referred to as S, N, M, and H).

*D* OK, we have this pie [chart] and we plan to use it [See Figure 5]. Everybody agrees?

Students Yes, yes.

- *D* Let's see what should we say here? Actually we see that ... 60 percent of ...
- A 60 percent of the workers are under the average wage [4500 IS]. Now, by adding 12 percent there are far fewer [workers under the national average].
- *S* OK, but I have a proposal, that brings almost everybody above the average wage. If we add 1000 shekel to the 49 workers, who are under the average ...
- *N* It's impossible. Can't you understand that?
- *S* This [my proposal] will leave us with 1000 shekel, that can be divided among the other workers, who are over [the average].
- A Then each of them will get exactly five shekel! ...
- *M* But we don't have any chance to win this way.
- *D* What is the matter with you? We'll have a revolt in our own ranks. Do you want that to happen at the final debate?
- *S* Anyway, this is my opinion! If there are no better proposals ...
- *D* Of course there are: a rise of 12 percent on each salary [excluding the managers] ...
- *H* OK. Show me by how much will your proposal reduce the 60 percent.
- *N* I am printing now an amazing proposal—everybody will be above the [national] average: No worker will be under the average wage! This needs a considerable cut in the managers' salaries ...



Figure 5. The "workers" description of the current salary distribution.

In this exchange, three different proposals for the alteration of the salary structure were presented. The first, offered by A and D, suggested an increase of 12 percent for all workers but the managers' salaries remained unchanged. The second proposal, originated by S, suggested an equal (1000 IS) increase for each of the 49 workers earning less than the national average (4350 IS), the small remainder to be divided among the other workers. Again the managers' salaries remained

unchanged. The third proposal, presented by N, suggested a considerable cut in managers' salaries, and an increase for all workers under the national average, to bring them above the average.

Central to students' actions and motives is the stand to be taken by the workers. For example, Figure 5 is grouped to emphasize the large proportion of salaries below the printers' national average. Moreover, the workers' explanations for choosing representative measures and graphical displays emerged from their stand in the dispute. Taking a stand also made students check their methods, arguments, and conclusions with extreme care. They felt it natural to face criticism and counterarguments made by peers and teacher, and to answer them.

These observations suggest that students' reasoning about data as well as their interactions with data were strongly affected by the design of the problem situation, which includes *taking a stand*. The students were able to:

- Deal with a complex situation and the relevant statistical concepts (averages, percentages, charts, etc.).
- Select among measures of center, in relation to looking at graphs, which is an important component of EDA reasoning.
- Use critical arguments to confront conflicting alternatives.
- Use statistical procedures and concepts with a purpose and within a context, to solve problems, relying heavily on visual representations and computer.
- Demonstrate involvement, interest, enthusiasm, and motivation in their learning.
- Create their own products (proposals and their representations).

# DISCUSSION

The two case studies focused on students' reasoning about data analysis as they started to develop views (and tools to support them) that are consistent with the use of EDA. Sociocultural and cognitive perspectives will now be considered in a detailed analysis of the case studies. The sociocultural perspective focuses on learning (of a complex domain, such as EDA) as the adoption of the viewpoint of a community of experts, in addition to learning skills and procedures. Thus, this study looked at learning as an enculturation process with two central components: students engaged in doing, investigating, discussing and making conclusions; and teachers engaged in providing role models by being representatives of the culture their students are entering through timely interventions. The cognitive perspective focuses on the development and change in students' conceptions and the evolution of their reasoning. Learning is perceived as a series of interrelated actions by the learner to transform information to knowledge—such as collecting, organizing, and processing information—to link it to previous knowledge and provide interpretations (Davis, Maher, & Noddings, 1990).

It is not easy to tease out the two perspectives for this analysis. Conceptions and reasoning evolve within a purposeful context in a social setting. On the other hand,

developing an expert point of view, and interacting with peers or with a teacher, implies undergoing mental actions within specific tasks related to complex ideas. These actions over time are a central part of the meaningful experience within which the culture of the field is learned and the reasoning is developed. These perspectives contribute to the analysis of the data, which revealed the following factors in the process of developing students' reasoning about data in the EDA environment.

## The Role of Previous Knowledge

One of the strongest visible pieces of knowledge A and D applied and repeatedly referred to was the difference between single pairs of data, which came from their practices in the algebra curriculum. This background knowledge played several roles. On the one hand, it gave these students the differences lens, which conditioned most of what they were able to conclude for quite a while. On the other hand, looking at differences helped them to refocus their attention from "pure" pointwise observations toward more global conclusions (that the differences are not constant). Also, looking at differences helped the students, in implicit and subtle ways, to start getting accustomed to a new domain in which data do not behave in the deterministic way that the students were used to in algebra, in which regularities are captured in a single exact formula.

A and D's focus on the differences served more than one function in their learning. It was invoked and applied not only when they were asked to look for patterns in the data but also in a very fruitful way when they spontaneously evaluated the results of rescaling the graph. There, they used the differences in order to judge the extent to which the re-scaled graph matched their goal of designing a graph to support a certain claim about trends.

Thus *A* and *D*'s previous knowledge not only conditioned what they saw sometimes limiting them—but also, on other occasions, empowered them. Moreover, their previous knowledge served new emerging purposes, as it evolved in the light of new contextual experiences. In conclusion, this analysis illustrates the multifaceted and sometimes unexpected roles prior knowledge may play, sometimes hindering progress and at other times advancing knowledge in interesting ways.

# Moving from a Local-Pointwise View toward a Flexible Combination of Local and

#### Global Views

In the first case study, A and D persistently emphasized local points and adjacent differences. Their views were related to their "history" (i.e., previous background knowledge about regularities with linear relationships in algebra). The absence of a precise regularity in a set of statistical data (understanding variability) was their first difficulty. When they started to adopt the notion of trend (instead of the regular algebraic pattern expected), they were still attentive to the prominence of "local deviations." These deviations kept them from dealing more freely with global views of data. Later on, it was precisely the focus on certain pointwise observations (for

example, the place and deletion of one outlying point) that helped them to direct their attention to the shape of the (remaining) graph as a whole. During the scaling process, A and D looked at the graph as a whole; but rather than focusing on the trends, they discussed its relative locations under different scales. Finally, when they used the scaling and had to relate to the purpose of the question (support of claims in the journalists' debate), they seemed to begin to make better sense of trends.

It is interesting to note that the local pointwise view of data sometimes restrained the students from seeing globally, but in other occasions it served as a basis upon which the students started to see globally. In addition, in a certain context, even looking globally indicated different meanings for the students than for an expert (i.e., noting the position of the graph rather than noticing a trend).

# Appropriation: A Learning Process That Promotes Understanding

The data show that most of the learning took place through dialogues between the students themselves and in conversations with the teacher. Of special interest were the teacher's interventions, at the students' request (additional examples of such interventions are described in Ben-Zvi & Arcavi, 2001). These interventions, though short and not necessarily directive, had catalytic effects. They can be characterized in general as "negotiations of meanings" (in the sense of Yackel & Cobb, 1996). More specifically, they are interesting instances of *appropriation* as a nonsymmetrical, two-way process (in the sense of Moschkovich, 1989). This process takes place, in the *zone of proximal development* (Vygotsky, 1978, p. 86), when individuals (expert and novices, or teacher and students) engage in a joint activity, each with their own understanding of the task. Students take actions that are shaped by their understanding; the teacher "appropriates" those actions—into her own framework—and provides feedback in the form of her understandings, views of relevance, and pedagogical agenda. Through the teacher's feedback, the students start to review their actions and create new understandings for what they do.

In this study, the teacher appropriated students' utterances with several objectives: to legitimize their directions, to redirect their attention, to encourage certain initiatives, and implicitly to discourage others (by not referring to certain remarks). The students appropriate from the teacher a reinterpretation of the meaning of what they do. For example, they appropriate from her answers to their inquiries (e.g., what *trend* or *interesting phenomena* may mean), from her unexpected reactions to their request for explanation (e.g., "How did you flatten the graph?"), and from inferring purpose from the teacher's answers to their questions (e.g., "We are supposed to look at what?").

Appropriation by the teacher (to support learning) or by the students (to change the sense they make of what they do) seems to be a central mechanism of enculturation. As shown in this study, this mechanism is especially salient when students learn the dispositions that accompany using the subject matter (data analysis) rather than its skills and procedures.

### Curriculum Design to Support Reasoning about Data

The example described in the second case study illustrates how curriculum design can take into account new trends in subject matter (EDA)—its needs, values, and tools—as well as student reasoning. Staging and encouraging students to *take sides* pushed them toward levels of reasoning and discussion that have not been observed in the traditional statistics classroom. They were involved in selecting appropriate statistical measures, rather than just calculating them, and in choosing and designing graphs to best dispaly their views. They showed themselves able to understand and judge the complexities of the situation—engaged in preparing a proposal that in their view was acceptable, rational, and just—and were able to defend it.

Furthermore, students realized that data representations could serve *rhetorical functions*, similar to their function in the work of statisticians, who select data, procedures, tools, and representations that support their perspective. Thus, the development of students' reasoning about data is extended beyond the learning of statistical mathods and concepts, to involve students in "doing" statistics in a realistic context.

# IMPLICATIONS

The learning processes described in this chapter took place in a carefully designed environment. It is recommended that similar environments be created to help students develop their reasoning about data analysis. The essential features of such learning environments include

- A curriculum built on the basis of EDA as a sequence of semi-structured (yet open) leading questions within the context of extended meaningful problem situations (Ben-Zvi & Arcavi, 1998)
- Timely and nondirective interventions by the teacher as representative of the discipline in the classroom (cf., Voigt, 1995)
- Computerized tools that enable students to handle complex actions (change of representations, scaling, deletions, restructuring of tables, etc.) without having to engage in too much technical work, leaving time and energy for conceptual discussions

In learning environments of this kind, students develop their reasoning about data by meeting and working with, from the very beginning, ideas and dispositions related to the culture of EDA. This includes making hypotheses, formulating questions, handling samples and collecting data, summarizing data, recognizing trends, identifying variability, and handling data representations. Skills, procedures and strategies (e.g., reading graphs and tables, rescaling) are learned as integrated in the context and at the service of the main ideas of EDA.

It can be expected that beginning students will have difficulties of the type described when confronting the problem situations posed by the EDA curriculum. However, what A and D experienced is an integral and inevitable component of their meaningful learning process with long-lasting effects (cf., Ben-Zvi 2002). These results suggest that students should work in environments such as the one just described, which allows for:

- Students' prior knowledge to be engaged in interesting and surprising ways—possibly hindering progress in some instances but making the basis for construction of new knowledge in others
- Many questions to be raised—some will either make little sense to them, or, alternatively, will be reinterpreted and answered in different ways than intended
- Students' work to be based on partial understandings, which will grow and evolve

This study confirmed that even if students do not make more than partial sense of the material with which they engage, appropriate teacher guidance, in-class discussions, peer work and interactions, and more importantly, ongoing cycles of experiences with realistic problem situations, will slowly support the building of meanings and the development of statistical reasoning.

Multiple challenges exist in the assessment of outcomes of students' work in such a complex learning environment: the existence of multiple goals for students, the mishmash between the contextual (real-world) and the statistical, the role of the computer-assisted environment, and the group versus the individual work (Gal & Garfield, 1997). It is recommended that extended performance tasks be used to assess students' reasoning about data, instead of traditional tests that focus on definitions and computation. Performance tasks should be similar to those given to students during the learning activities (e.g., open-ended questions, "complete" data investigations), allowing students to work in groups and use technological tools.

In EDA learning environments of the kind described in these case studies, teachers cease to be the dispensers of a daily dose of prescribed curriculum and must respond to a wide range of unpredictable events. They can play a significant role in their interactions with students by encouraging them to employ critical reasoning strategies and use data representations to search for patterns and convey ideas; expanding and enriching the scope of their proposed work; and providing reflective feedback on their performance. Thus our challenge is to assist statistics educators in their important role of mentors and mediators, or the *enculturators*.

Given that EDA is a challenging topic in statistics education and is part of the mathematics curriculum in many schools today, it is important that teaching efforts be guided not only by systematic research on understanding the core ideas in data analysis but also by how reasoning about data analysis develops. Without this research and the implementation of results, statistics classes will continue to teach graphing and data-collection skills that do not lead to the ability to reason about data analysis.

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Many research questions need to be addressed, including those pertaining to the development of students' understanding and reasoning (with the assistance of technological tools), the student-teacher and student-student interactions within open-ended data investigation tasks, the role of enculturation processes in learning, and the impact of learning environments similar to those described here. The refinement of these ideas, and the accumulation of examples and studies, will contribute to the construction of an EDA learning and instruction theory.

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