Chapter 17

RESEARCH ON STATISTICAL LITERACY, REASONING, AND THINKING: ISSUES, CHALLENGES, AND IMPLICATIONS

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INTRODUCTION

The collection of studies in this book represents cutting-edge research on statistical literacy, reasoning, and thinking in the emerging area of statistics education. This chapter describes some of the main issues and challenges, as well as implications for teaching and assessing students, raised by these studies. Because statistics education is a new field, taking on its own place in educational research, this chapter begins with some comments on statistics education as an emerging research area, and then concentrates on various issues related to research on statistical literacy, reasoning, and thinking. Some of the topics discussed are the need to focus research, instruction, and assessment on the big ideas of statistics; the role of technology in developing statistical reasoning; addressing the diversity of learners (e.g., students at different educational levels as well as their teachers); and research methodologies for studying statistical reasoning. Finally, we consider implications for teaching and assessing students and suggest future research directions.

STATISTICS EDUCATION AS AN EMERGING RESEARCH AREA

Statistics and statistics education are relatively new disciplines. Statistics has only recently been introduced into school curricula (e.g., NCTM, 2000) and is a new academic major at the college level (Bryce, 2002). In the United States, the NCTM standards (2000) recommend that instructional programs from pre-kindergarten through grade 12 focus more on statistical reasoning. The goals of their suggested statistics curriculum include

- Enable all students to formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

- Select and use appropriate statistical methods to analyze data.
- Develop and evaluate inferences and predictions that are based on data.
- Understand and apply basic concepts of probability.

At the university level, statistics is taught at undergraduate as well as graduate levels across many disciplines. The students taking statistics at these levels may be preparing to be future "users" or "producers" of statistics in different fields of application (e.g., sciences, technology, industry, and medicine), or future statisticians or statistics teachers. Over the last 20 years there has been a steady increase in the numbers of statistics courses taught, to fulfill the growing demand for students and professionals who can use and understand statistical information.

Although the amount of statistics instruction at all levels is growing at a fast pace, the research to support statistics instruction is proceeding at a much slower rate. The research literature in statistics education is not well known; therefore, it is not often valued or utilized by statisticians, schools, or the immense number of other fields that use statistics (Joliffe, 1998). In fact, researchers in this area argue that the field still needs to define what research in statistics education is—not only to achieve academic recognition, but to convince others of its validity as a research discipline (Batanero, Garfield, Ottaviani, & Truran, 2000).

Unlike other research areas, the research studies on teaching and learning statistics have been conducted in, and influenced by, several different disciplines, each with its own perspectives, literatures, methodology, and research questions. For example, much of the early research was conducted by psychologists, often focusing on conceptions of chance and randomness (e.g., Piaget & Inhelder, 1975; Fischbein, 1975; and Kahneman, Slovic, & Tversky, 1982). Psychologists' dominant effort was to identify, through observations or paper and pencil tests, ways in which people make judgments of chance. Many researchers (for example, Kahneman et al., 1982; Konold, 1989) identified widespread errors in reasoning, finding that people tend to use nonstatistical heuristics to make judgments or decisions regarding chance events. By the end of the 1980s, there was strong evidence that many adults are unable to deal competently with a range of questions that require probabilistic thinking.

In the 1980s and 1990s, many researchers in mathematics education, motivated by the inclusion of statistics and probability in the elementary and secondary mathematics curriculum, began to explore students' understanding of ideas related to statistics and data analysis (e.g., Russell & Mokros, 1996; Mokros & Russell, 1995; Rubin, Bruce, & Tenney, 1991; Shaughnessy, 1992). These researchers found the mathematics education theoretical frameworks and methodologies relevant for research in statistics education (see, for example, Kelly & Lesh, 2000). During this same time period, several educational psychologists explored students' attitudes and anxiety about statistics in an attempt to predict success in statistics courses (e.g., Wisenbaker & Scott, 1997; Schau & Mattern, 1997), while cognitive psychologists examined ways to help students and adults correctly use statistical reasoning (e.g., Fong, Krantz, & Nisbett, 1986; Nisbett, 1993; Sedlmeier, 1999). A more recent group of researchers is emerging from the growing number of statisticians who are focusing their scholarship on educational issues (e.g., Chance, 2002; Lee, Zeleke, & Wachtel, 2002; Wild, Triggs, & Pfannkuch, 1997). Some of these researchers looked at particular classroom interventions and their impact on learning outcomes or developed models for teaching and for experts' statistical thinking.

What has been missing in the past few decades is a coordination of the research across the different disciplines described earlier, and a convergence of methods and important research questions. Without this coherence, it is hard to move the field forward and to build on the results by linking research to teaching. One example of an effort to coordinate the research across the different disciplines is the book edited by Lajoie (1998), which addresses issues of statistical content, learner needs, instructional methods, and assessment goals. It was the outcome of the coordinated work of statisticians, mathematics educators, and psychologists, who focused on formulating a research agenda for K–12 statistics education.

The International Research Forums on Statistical Reasoning, Thinking, and Literacy (SRTL-1, Israel; SRTL-2, Australia; and SRTL-3, USA) have been another important effort to achieve this goal, by bringing together an international group of researchers from across these disciplines to share their findings, discuss their methods, and generate important issues and research questions. Another goal of these research forums has been to make explicit connections to teaching practice, something that researchers are often criticized for failing to address.

RESEARCH ON STATISTICAL LITERACY, REASONING, AND THINKING

Although statistics is now viewed as a unique discipline, statistical content is most often taught in the mathematics curriculum (K–12) and in departments of mathematics (tertiary level). This has led to exhortations by leading statisticians, such as Moore (1998), about the differences between statistics and mathematics (see Chapter 4). These arguments challenge statisticians and statistics educators to carefully define the unique characteristics of statistics, and in particular, the distinctions between statistical literacy, reasoning, and thinking. We provided summaries of these arguments and related research in the early chapters of this book (see Chapters 1 through 4).

The authors of chapters in this book represent the growing network of researchers from SRTL-1 and SRTL-2 who are interested in statistical literacy, reasoning, and thinking, and who have been trained in the different disciplines (e.g., mathematics education, cognitive and educational psychology, and statistics). Many of the chapters describe collaborative studies, some including researchers from different disciplines (e.g., Chapters 2, 13, and 14). It may seem strange, given the quantitative nature of statistics, that most of the studies in this book include analyses of qualitative data, particularly videotaped observations or interviews. We have found that sharing these videos and their associated transcripts allows us to better present and discuss the important aspects of our work, as well as to solicit useful feedback from colleagues. Further discussion of methodological issues is provided later in this chapter.

The topics of the research studies presented in this book reflect the shift in emphasis in statistics instruction, from statistical techniques, formulas, and procedures to developing statistical reasoning and thinking. The chapters on individual aspects of reasoning focus on some core ideas of statistics, often referred to as the 'big ideas." Increasing attention is being paid in the educational research community to the need to clearly define and focus both research and instruction, and therefore, assessment, on the big ideas of a discipline (Bransford, Brown, & Cocking, 2000; Wiggins, 1998). We offer a list and description of the big ideas of statistics in the following section.

FOCUSING ON THE BIG IDEAS OF STATISTICS

The topics of the chapters in this book (e.g., data, distribution, averages, etc.) focus on some of the big ideas in statistics that students encounter in their educational experiences in elementary, secondary, or tertiary classes. Although many statistics educators and researchers today agree that there should be a greater focus on the big ideas of statistics, little has been written about what these ideas are. Friel (in press) offers a list similar to the one we provide here:

- **Data**—the need for data; how data represent characteristics or values in the real world; how data are obtained; different types of data, such as numbers, words, and so forth.
- **Distribution**—a representation of quantitative data that can be examined and described in terms of shape, center, and spread, as well as unique features such as gaps, clusters, outliers, and so on.
- **Trend**—a signal or pattern we are interested in. It could be a mean for one group, the difference of means for comparing two groups, a straight line for bivariate data, or a pattern over time for time-series data.
- **Variability**—the variation or noise around a signal for a data set, such as measurement error. Variability may also be of interest in that it helps describe and explain a data set, reflecting natural variation in measurements such as head sizes of adult men.
- **Models—**an ideal that is sometimes useful in understanding, explaining, or making predictions from data. A model is useful if it 'fits" the data well. Some examples of models are the normal curve, a straight line, or a binomial random variable with probability of 0.5.
- **Association**—a particular kind of relationship between two variables; information on one variable helps us understand, explain, or predict values of the other variable. Association may be observed between quantitative or categorical variables. This also includes being able to distinguish correlation from causality.
- **Samples and sampling**—the process of taking samples and comparing samples to a larger group. The sampling process is important in obtaining a representative sample. Samples are also used to generate theory, such as simulating sampling distributions to illustrate the Central Limit Theorem.
- **Inference—**ways of estimating and drawing conclusions about larger groups based on samples. Utts (2003) elaborates that this includes being able to differentiate between practical and statistical significance as well as knowing

the difference between finding "no effect" versus finding "no significant effect."

When we examine much of statistics instruction, it is not always clear how these big ideas are supposed to be presented and developed. In most statistics classroom instruction, the emphasis is on individual concepts and skills, and the big ideas are obscured by the focus on procedures and computations. After one topic has been studied, there is little mention of it again, and students fail to see how the big ideas are actually providing a foundation for course content and that they underlie statistical reasoning. For example, students may focus on how to compute different measures of center or variability without fully understanding the ideas of center and spread and their relationships to other big ideas, such as data and distribution. Later in their studies, students may fail to connect the idea of center and spread of sampling distributions with the ideas of center and spread in descriptive statistics. Or, when studying association, students may lose track of how center and spread of each variable are embedded in looking at bivariate relationships.

Major challenges that teachers face include not only finding ways to go beyond the individual concepts and skills, but leading students to develop an understanding of the big ideas and the interrelations among them. Such an approach will enable teachers to make the big ideas explicit and visible, throughout the curriculum. For example, Cobb (1999) suggests that focusing on distribution as a multifaceted end goal of instruction in seventh grade might bring more coherence in the middle school statistics curriculum and empower students' statistical reasoning. Bakker and Gravemeijer (Chapter 7) propose to focus instruction on the informal aspects of shape. Other illustrations of the need to focus on the big ideas of statistics and how to do it can be found in various chapters of this book: data (Chapter 6), center (Chapter 8), variability (Chapter 9), covariation (Chapter 10), and sampling (Chapters 12 and 13). It has been suggested that the use of technology-assisted learning environments can support—in many ways—students' construction of meanings for the big ideas of statistics (e.g., Garfield & Burrill, 1997).

THE ROLE OF TECHNOLOGY IN DEVELOPING STATISTICAL REASONING

Many of the chapters in this book mention the use of technology in developing statistical reasoning. This is not surprising, given how the discipline of statistics has depended on technology and how technology has been driving change in the field of statistics. Although there are many technological tools available, including graphing calculators, computers, and the World Wide Web, there is still a lack of research on how to best use these tools and how they affect student learning.

The interaction of technology with efforts to redefine both content and instruction in statistics in the K–12 curriculum provides a variety of strategies for *teaching* statistics and, at the same time, offers new ways of *doing* statistics (Garfield & Burrill, 1997). Today, computers, software, and the Internet are essential tools for instruction in statistics (Friel, in press).

Ben-Zvi (2000) describes how technological tools may be used to help students actively construct knowledge, by "doing" and "seeing" statistics, as well as to give students opportunities to reflect on observed phenomena. He views computers as cognitive tools that help transcend the limitations of the human mind. Therefore, technology is not just an amplifier of students' statistical power, but rather a reorganizer of students' physical and mental work. The following types of software, which are described in this book, are good examples of such tools:

- *Commercial statistical packages* for analyzing data and constructing visual representations of data such as spreadsheets (Excel©, Chapter 6), or data analysis programs (Statgraphics©, Chapter 11) that offer a variety of simultaneous representations that are easily manipulated and modified, as well as simulation of different distributions.
- *Educational data analysis tools* (Fathom©, Chapter 15) are intended to help students develop an understanding of data and data exploration. They support in-depth inquiry in statistics and data analysis through powerful statistical and plotting capabilities that give the user greater overall control in structuring and representing data (Friel, in press). Fathom also allows plotting functions, creating animated simulations, and has a "dragging" facility that dynamically updates data representations. This helps reveal the invariant phenomenon and the relationships among representations.
- *Web- or computer-based applets* were developed to demonstrate and visualize statistical concepts. Applets are typically small, web-based computer programs that visually illustrate a statistical concept by letting the user manipulate and change various parameters. The Minitools (Chapters 7 and 16), a particular type of applet, were designed to support an explicit "learning trajectory" to develop an understanding of a particular graph and its link to the data on which it is based.
- *Stand-alone simulation software*, such as Sampling SIM (Chapter 13), which was developed to provide a simulation of sampling distributions, with many capabilities allowing students to see the connections between individual samples, distributions of sample means, confidence intervals, and *p*-values.

The last three tools on this list (Fathom, Minitools, and Sampling SIM) were designed based on ideas about what students need to see and do in order to develop a conceptual understanding of abstract statistical concepts as well as develop the kinds of attitudes and reasoning required for analyzing data. Although these three tools were developed to improve student learning, Bakker (2002) distinguished between route-type software––small applets and applications, such as the Minitools that fit in a particular learning trajectory; and landscape-type software––larger applications, such as Fathom and TinkerPlots, that provide an open landscape in which teachers and students may freely explore data.

The increasing use of Internet and computer-mediated communication (CMC) in education has also influenced statistics education. Although not the focus of chapters in this book, there are numerous Internet uses in statistics classes that support the development of students' statistical reasoning. For example, data sources in downloadable formats are available on the Web to support active learning of exploratory data analysis. They are electronically available from data-set archives, government and official agencies, textbook data, etc. An additional example is the use of CMC tools, such as online forums, e-mail, and so forth to create electronic communities that support students' learning in face-to-face or distance learning.

It is important to note that despite its role in helping students learn and do statistics, technology is not available in all parts of the world, and not even in all classrooms in the more affluent countries. The research studies in this book address different instructional settings with and without the use of technology, as well as diverse types of students who are learning statistics at all levels.

DIVERSITY OF STUDENTS AND TEACHERS

With the growing emphasis on statistical literacy, reasoning, and thinking, statistics education research must address the diversity of students in statistics courses by considering issues of continuity (when to teach what), pedagogy (how to approach the content and develop desired learning outcomes), priority (prioritizing and sequencing of topics), and diversity (students' educational preparation and background, grade and level). For example, little attention has been given to the issue of when and how a new statistical idea or concept can be presented to students, or to the question of sequencing statistical ideas and concepts along the educational life span of a student.

The individual research studies in this book partially address such issues, but as a group reflect the diversity of students (and teachers) who learn and know statistics. The widest "student" population is addressed in research about statistical literacy, which includes school students through adults. Gal (Chapter 3) underscores the importance of statistical literacy education for all present and future citizens to enable them to function effectively in an information-laden society. The goal of statistical literacy research is to identify the components of literacy, to find ways to equip all citizens with basic literacy skills—such as being able to critically read the newspaper or evaluate media reports.

The students observed by Ben-Zvi (Chapter 6) as well as Bakker and Gravemeijer (Chapter 7) were high-ability students. The forms of reasoning exhibited by some of these students are to some extent unique to the specific settings and circumstances. However, these studies describe some important teaching and learning issues and how the reasoning might develop in other types of students. They also suggest meaningful and engaging activities such as making predictions graphs without having data and using software tools that support specific statistical ways of reasoning. The instructional suggestions in some chapters require establishing certain socio-mathematical (statistical) norms and practices (Cobb & McClain, Chapter 16), use of suitable computer tools, carefully planned instructional activities, and skills of the teacher to orchestrate class discussions.

Ben-Zvi (Chapter 6) and Mickelson and Heaton (Chapter 14) describe the teachers in their studies as above average in pedagogical and statistical knowledge and skills. It is likely that the role of "average" elementary and middle school teachers, normally not trained in statistics instruction, would be quite different. Teachers need careful guidance to teach such a new and complex subject. Hence, more studies are needed that explore how to equip school teachers at all levels with appropriate content knowledge and pedagogical knowledge, and to determine what kind of guidance they need to successfully teach these topics.

RESEARCH METHODOLOGIES TO STUDY STATISTICAL REASONING

The chapters in this book reveal a variety of research methods used to study statistical literacy, reasoning, and thinking. Ben-Zvi (Chapter 6), and Mickelson and Heaton (Chapter 14) use a case study approach in natural classroom settings to study one or two cases in great detail. Batanero, Tauber, and Sánchez (Chapter 11) use a semiotic approach to analyze students' responses to open-ended questions on an exam. Chance, delMas, and Garfield (Chapter 13) use collaborative classroom research to develop software and build a model of statistical reasoning. Their research is implemented in their own classes and with their students, using an iterative cycle to study the impact of an activity on students' reasoning as they develop their model. Their method of classroom research is similar to the classroom teaching experiment used in the studies by Bakker and Gravemeijer (Chapter 7) and Cobb and McClain (Chapter 16), who refer to this method as design experiment (described by Lesh, 2002). Watson (Chapter 12) uses a longitudinal approach to study children's development of reasoning about samples.

As mentioned earlier, videotaped classroom observations and teacher or student interviews were included in most studies as a way to gather qualitative data. We have found in analyzing these videos, that observing students' verbal actions as well as their physical gestures helps us better understand students' reasoning and the socio-cultural processes of learning. Other sources of qualitative data were students' responses to open-ended questions, field notes of teachers and researchers, and samples of students' work (e.g., graphs constructed, statistics projects).

Makar and Confrey (Chapter 15) combine qualitative data with quantitative data on teachers' statistical reasoning. Pre- and posttests of statistical content knowledge provided the main source of quantitative data for their study, while videotaped interviews were transcribed and then analyzed using grounded theory (Strauss & Corbin, 1998). A few other studies also include some quantitative data in the context of student assessment, for example, Reading and Shaughnessy (Chapter 9), Moritz (Chapter 10), Batanero et al. (Chapter 11), and Watson (Chapter 12).

It may seem surprising that few statistical summaries are actually included in these studies, given that the subject being studied by students or teachers is statistics. And it may seem surprising that the research studies in this book are not traditional designed experiments, involving control groups compared to groups that have received experimental treatment, the gold standard of experimental design. However, statistics education tends to follow the tradition of mathematics and science education, in using mostly qualitative methods to develop an understanding of the nature of students' thinking and reasoning, and to explore how these develop (see Kelly & Lesh, 2000). Perhaps after more of this baseline information is gathered and analyzed, the field will later include some small, experimental studies that allow for comparisons of particular activities, instructional methods, curricular trajectories, types of technological tools, or assessments.

Before we reach this stage of research, we need to further study the long-lasting effects of instruction on students' reasoning, and to continue the exploration of models of conceptual change and development. These models will be based on careful examination and analyses of how reasoning changes, either over an extensive period of time (as in longitudinal studies) or during periods of significant transition (as in some clinical interviews or classroom episodes).

IMPLICATIONS FOR TEACHING AND ASSESSING STUDENTS

In the three chapters that focus on the topics of statistical thinking (Chapter 2), statistical literacy (Chapter 3), and statistical reasoning (Chapter 4), each author, or pair of authors, recommends that instruction be designed to explicitly lead students to develop these particular learning outcomes. For example, Pfannkuch and Wild (Chapter 2) discuss the areas to emphasize for developing statistical thinking, Gal (Chapter 3) describes the knowledge bases and dispositions needed for statistical literacy, and delMas (Chapter 4) describes the kinds of experiences with data that should lead to statistical reasoning.

One important goal of this book is provide suggestions for how teachers may build on the research studies described to improve student learning of statistics. Although most teachers do not typically read the research related to learning their subject matter content, we encourage teachers of statistics at the elementary, secondary, and tertiary level to refer to chapters in this book for a concise summary of research on the different areas of reasoning. These studies provide ideas not only about the types of difficulties students have when learning particular topics, so that teachers may be aware of where errors and misconceptions might occur, but also what to look for in their informal and formal assessments of students learning. In addition, these studies provide valuable information regarding the type of statistical reasoning that can be expected at different age levels. The models of cognitive development in statistical reasoning documented in Chapter 5 enable teachers to trace students' individual and collective development in statistical reasoning during instruction. Because the cognitive models offer a coherent picture of students' statistical reasoning, they can provide a knowledge base for teachers in designing and implementing instruction.

These research studies include details on the complexity of the different statistical topics, explaining why they are so difficult for students to learn. As several authors stress, it is important for teachers to move beyond a focus on skills and computations, and the role of teacher as the one who delivers the content. Instead, the role of teacher suggested by the authors of these chapters is one of providing a carefully designed learning environment, appropriate technological tools, and access to real and interesting data sets. The teacher should orchestrate class work and discussion, establish socio-statistical norms (see Cobb and McClain, Chapter 16) and provide timely and nondirective interventions by the teacher as representative of the discipline in the classroom (e.g., Voigt, 1995). The teacher should be aware not only of the complexities and difficulty of the concepts, but of the desired learning goals—such as what good statistical literacy, reasoning, and thinking look like—so that assessments can be examined and compared to these goals. The teachers need to be comfortable with both the content and tools, and with the process of data analysis.

The chapters in this book stress that students need to be exposed to the big ideas and their associated reasoning in a variety of settings, through a course or over several years of instruction. The authors make many suggestions about how technology can be used to help students develop their reasoning, and suggest that students be prodded to explain what they see and learn when using these tools as a way to develop their reasoning.

Many of the authors present some types of learning activities and data sets that teachers can use in their classes at different school levels. They suggest that regardless of the activity used, teachers can find ways to observe their students carefully to see how their reasoning is affected by the activities. Teachers should also avoid assuming that students have learned the material merely because they have completed an activity on that topic. Finally, teachers are encouraged to apply the research tools on their classes, and to use the information gathered to continually revise and improve their activities, materials, and methods. We believe that it is better to learn a few concepts in depth, rather than trying to cover every topic. If this can be done in a systematic way, then more topics might be covered over a span of grades, rather than in one single grade level.

We agree with the many educators who have called for classroom instruction to be aligned with appropriate methods of assessment, which are used as a way to make reasoning visible to teachers as well as to students. Assessment should be used for formative as well as summative purposes, and it should be aligned with learning goals. In most cases, a type of performance assessment seems to best capture the full extent of students' statistical reasoning and thinking (Gal & Garfield, 1997; Garfield & Gal, 1999).

We suggest that it is often helpful to start by considering the types of assessment that are appropriate to measure the desired learning outcomes, and to work backward, thinking about instruction and activities that will lead to these goals (see Wiggins, 1998). Then assessment data gathered from students can be used to evaluate the extent to which these important learning goals (e.g., developing statistical reasoning) have been achieved.

FUTURE DIRECTIONS AND CHALLENGES

Given the importance of the learning outcomes described in this book, statistical literacy, reasoning and thinking, it is crucial that people working in this area use the same language and definitions when discussing these terms. Similarly, some standard goals for each outcome should be agreed upon and used in developing educational materials and curricula, designing assessments, preparing teachers' courses, and conducting future research.

Because the field of statistics education research is so new, there is a need for more research in all of the areas represented in this book. Studies need to be conducted in different educational settings, with different-aged students worldwide, and involving different educational materials and technological tools. As we continue to learn more about how different types of reasoning in statistics develop, we need to continue to explore cognitive developmental models, seeing how these apply to the different settings. There is also a need to validate these models, and to investigate how they may be used to promote reasoning, thinking, and literacy through carefully designed instruction.

There is a great need for assessment instruments and materials that may be used to assess statistical literacy, reasoning, and thinking. A set of accessible, high-quality instruments could be used in future evaluation and research projects to allow more comparison of students who study with different curricula or in different educational settings.

SUMMARY

This book focuses on one aspect of the "infancy" of the field of statistics education research, by attempting to grapple with the definitions, distinctions, and development of statistical literacy, reasoning, and thinking. As this field grows, the research studies in this volume should help provide a strong foundation as well as a common research literature. This is an exciting time, given the newness of the research area and the energy and enthusiasm of the contributing researchers and educators who are helping to shape the discipline as well as the future teaching and learning of statistics. We point out that there is room for more participants to help define and construct the research agenda and contribute to results. We hope to see many new faces at future gatherings of the international research community, whether at SRTL-4, or 5, or other venues such as the International Conference on Teaching Statistics (ICOTS), International Congress on Mathematical Education (ICME), and the International Group for the Psychology of Mathematics Education (PME).

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