SCAFFOLDING STUDENTS' INFORMAL INFERENCE AND ARGUMENTATION[1]

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ABSTRACT

This paper focuses on developing students' informal ideas of inference and argumentative skills. This topic is of current interest to many researchers and teachers of statistics. We study fifth graders' learning processes in an exploratory interdisciplinary learning environment that uses TinkerPlots to scaffold and extend students' statistical reasoning. The careful design of the learning trajectory based on growing samples heuristics coupled with the unique features of TinkerPlots were found instrumental in supporting students' multiplicative reasoning, aggregate reasoning, acknowledging the value of large samples, and accounting for variability. These processes were accompanied by greater ability to verbalize, explain and argue about data-based inferences. In the light of the analysis, a description of what it may mean to begin reasoning and arguing about inference by young students is proposed.

Keywords: EDA; Informal inference; Statistical reasoning; Argumentation.

INTRODUCTION

In the context of an interdisciplinary exploratory learning environment that uses the software *TinkerPlots* (Konold and Miller, 2005), we focus on developing students' informal ideas of inference. As new statistics courses and curricula are developed at all levels, a greater role for informal types of statistical inference rather than on formal methods of estimation and tests of significance is anticipated, introduced early, revisited often, and developed through use of simulation and technological tools. We also focus on argumentative activity that was found beneficial for knowledge building and evaluation of information in some conditions (Schwarz, Neuman, Gil and Ilya, 2003). In the following paragraphs, we briefly describe the theoretical underpinnings of the study, the design of the curriculum, and the type of results and implications that will be presented in ICOTS.

THEORETICAL BACKGROUND: INFERENCE AND ARGUMENTATION

Statistical inference is “the theory, methods, and practice of forming judgments about the parameters of a population, usually on the basis of random sampling” (Collins English Dictionary, 2000). There are two important themes in statistical inference: hypothesis testing and parameter estimation and two kinds of inference questions: generalizations (surveys) and comparison and cause (experiments). In general terms, the first is concerned with generalizing from a small sample to a larger population, while the second has to do with determining if a pattern in the data is due to cause and effect.

Most of what is usually done in statistics in primary level is part of the exploratory data analysis approach to data (EDA, Tukey, 1977). The emphasis is mostly on ways to uncover, display, and describe interesting patterns in data. Inferences are informal, based on what we see in the data, and

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apply only to the individuals and circumstances for which we have data in hand. The problem is that this deficient inferential reasoning may result in shaky conclusions and give students a wrong deterministic sense of statistics. Rudimentary statistical methods taught at primary level, such as tabulating and graphing data, can help students look for interesting patterns in simple data sets, but are not enough to take them beyond the data in hand. Our project concentrates on helping young students "to draw conclusions about a wider universe, taking into account that variation is everywhere and the conclusions are uncertain" (Moore, 2000, p. xxx).

Since formal inference ideas and techniques are beyond the reach of young students, an informal approach to teaching and learning is necessary. Rubin, Hammerman and Konold (2006) view "informal inference" as statistical reasoning that involves consideration of multiple dimensions: Properties of aggregates rather than properties of the individual cases themselves, signals and noise, various forms of variability, sample size, controlling for bias, and tendency (claims that are always true and those that are often or sometimes true). Research in statistics education has however demonstrated that students at all levels have many difficulties in reasoning about these dimensions (cf., Ben-Zvi & Garfield, 2004; Bakker et al., 2004).

Informal inference is closely related to argumentative activities. Deriving logical conclusions from data - whether formally or informally - is accompanied by the need to provide persuasive arguments based on data analysis. Argumentation refers to discourse for persuasion, logical proof, and evidence-based belief, and more generally, discussion in which disagreements and reasoning are presented (Kirschner, Buckingham Shum and Carr, 2003). Toulmin's suggestion (2003) that any argument consists of data, warrant, backing, qualifier, reservation and claim, was used by others (cf., Glassner and Schwarz, 2005) to develop and study students' argumentative skills.

In sum, integration and cultivation of informal inference and informal argumentation seem to be essential in constructing students' statistical knowledge and reasoning in rich learning contexts. This view is supported by Abelson (1995), who proposes two essential dimensions to informal argumentation: The act or process of deriving logical conclusions from data (inference), and providing persuasive arguments based on the data analysis (rhetoric and narrative). In the service of persuasive arguments based on data analysis, forceful rhetoric and effective narrative are combined. "In making her best case, the investigator must combine the skills of an honest lawyer, a good detective, and a good storyteller" (ibid, p. 16).

RESEARCH QUESTION

These ideas formed the motivation to explore the possibilities for students in primary school - with little prior formal statistical knowledge - to develop an informal understanding of inference in argumentative rich contexts. The following research question is used to structure the current study and the analysis of data collected: How do fifth grade students begin to reason about informal inference in a rich and supportive EDA learning environment? The environment involves open-ended EDA investigations designed by growing samples instructional heuristic, peer collaboration and group discussions, use of *TinkerPlots*, and guidance by math and science teachers.

CONTEXT AND METHOD

We study the emergence of students' reasoning about informal inference using developmental research in an exploratory data analysis learning environment (part of the *Connections* project), supported by a carefully designed instructional trajectory and the use of *TinkerPlots*.

The Connections Learning Environment

The investigators, statistics and science educators and researchers from the University of Haifa, worked with primary school students (grades 4-6, ages 10-12) to study their evolving ideas of statistical reasoning in an interdisciplinary computerized environment. Students actively experienced some of the processes involved in experts' practice of data-based enquiry by working on small data scenarios, investigated by peer collaboration and classroom discussions. Math and science teachers collaborated in guiding their students to actively model and analyze natural, sometimes complex, systems (for example, air pollution, water consumption) using statistical descriptions. The mathematics teachers guided their students through a series of genuine mini exploratory data analysis projects while the science teachers provided the scientific background and inquiry skills for the main research project (see Figure 1 for the main components of the project).

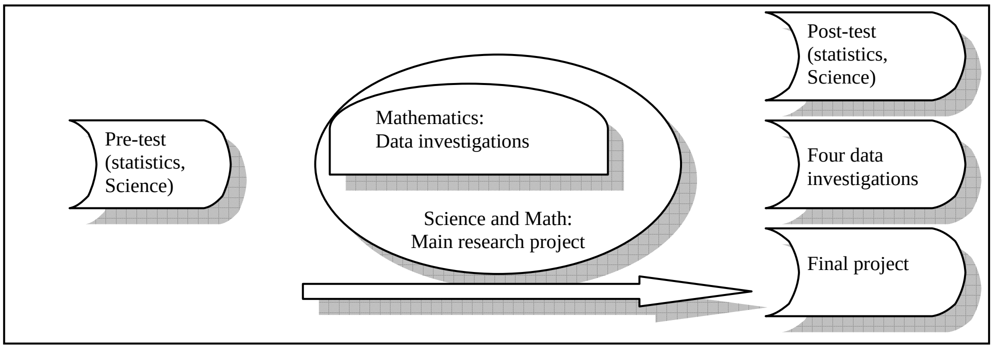

Figure 1: Outline of the *Connections* learning environment.

A central feature of the project is the use and the study of *TinkerPlots* (Konold and Miller, 2005), a statistical visualization tool that is designed to help students develop statistical reasoning and learn new ways of representing data. Students can begin using *TinkerPlots* without knowledge of conventional graphs or different data types, without thinking in terms of variables or axes. By progressively organizing their data (ordering, stacking, and separating data icons), students gradually organize data to answer their questions and actually design their own graphs.

Developmental Research

We carried out a developmental research (cf., Ben-Zvi, Garfield and Zieffler, 2006) to investigate students' construction of knowledge and improve the pre-formulated instructional design by checking and revising conjectures about the trajectory of learning for both the group and the individual students who compose the experiment population. The research was intensive (2-4 meetings a week, for 7-8 weeks) and somewhat invasive, in that each lesson is observed, videotaped and analyzed. The research had three stages: the preparation phase, the actual experimentation phase, and the retrospective analysis.

Participants

Three classes of fifth graders (n=75) in a science-focused magnet primary school in Haifa participated in the study. Most of the students come from affluent background and participated in EDA lessons with *TinkerPlots* in fourth grade. These previous encounters made them familiar with the software and basic statistical ideas.

Analysis

To assess students' learning we have used video recordings of all sessions, researcher's observations, interviews of selected students and teachers, and students' artifacts (notebooks and project reports). We also administered a math and science pre- and post-tests. These tests included 20 items (10 in each subject) taken from the released items of TIMSS (1995).

The analysis of the videotapes is based on interpretive microanalysis (see, for example, Meira, 1991): A qualitative detailed analysis of the protocols, taking into account verbal, gestural and symbolic actions within the situations in which they occurred. The goal of such an analysis is to infer and trace the development of cognitive structures and the sociocultural processes of understanding and learning. Two stages are used to validate the analysis, one within the researchers' team and one with additional researchers in education, who have no involvement with the data (triangulation in the sense of Schoenfeld, 1994). In both stages the researchers discuss, present, and advance and/or reject hypotheses, interpretations, and inferences about the students' cognitive structures. Advancing or rejecting an interpretation requires: (a) providing as many pieces of evidence as possible (including past and/or future episodes and all sources of data as described earlier) and (b) attempting to produce equally strong alternative interpretations based on the available evidence. The final report will include cases in which the two analyses are not in full agreement, and points of doubt or rejection are not refuted or resolved by iterative analysis of the data.

The Learning Trajectory in Math

Students collected and investigated real data about themselves and peer students and compared them to sample data generated from the UK CensusAtSchool data base (http://www.censusatschool.ntu.ac.uk/default.asp). Students were gradually introduced to increasing sample sizes: from one case (the student herself), to a very small sample of four familiar cases, to about 8, 16, 80, and 240 cases (each small sample is a subset of the larger samples). Following the *growing samples* instructional heuristic (Bakker & Gravemeijer, 2004), we encourage students to reason with stable features of variable processes, and compare their hypotheses regarding larger samples with observations generated by them from real data. We hoped that this process would not only help students get a good grip of the data at hand, but also support their reasoning about informal inference by observing aggregate features of distributions, identifying signals out of noise, accounting for the constraints of their inferences, and providing persuasive data-based arguments.

To collect real data, a 19 items questionnaire about gender and age, body measurements, home to school distance and time, computer and cellular phone ownership, etc. was used. Each fifth grader was assigned to collect data from randomly selected three students in grades 2, 4 and 6 as well as herself, and enter it to a *Tinkerplots* file. At this stage the class discussed issues of posing questions, census, measurement, sample and sampling.

Grouped in couples, students investigated first a small sample (n=8, four cases of each one of them), choosing questions they find interesting (for example association between height and arm span), proposing hypotheses, and continuously testing them by constructing and interpreting plots in *Tinkerplots*. Having generated several inferences, students were asked to hypothesize whether their inferences would hold in larger samples. Students were then grouped in quartets to test their hypotheses and discuss, confirm or refute their inferences based on a larger sample of about 16 cases (eight from each couple). The whole class discussed later the quartets' inferences using an overhead projector. Energized by peers' observations and fresh ideas the original couples depart back to the computer lab to further investigate their previous conjectures as well as new ideas, using this time a larger sample – about 80 students – generated by the whole class. This cycle of data analysis, informal inference, class presentation and reflection repeated twice on larger samples. At the final stage, students compared a sample of about 240 students from their school with 200 UK students (see the various stages in Figure 2).

At the same time, students worked in the science classroom on their main research project, analyzing natural systems using statistical descriptions. In a festive event, students presented and discussed their main research project in front of their peers, teachers and parents.

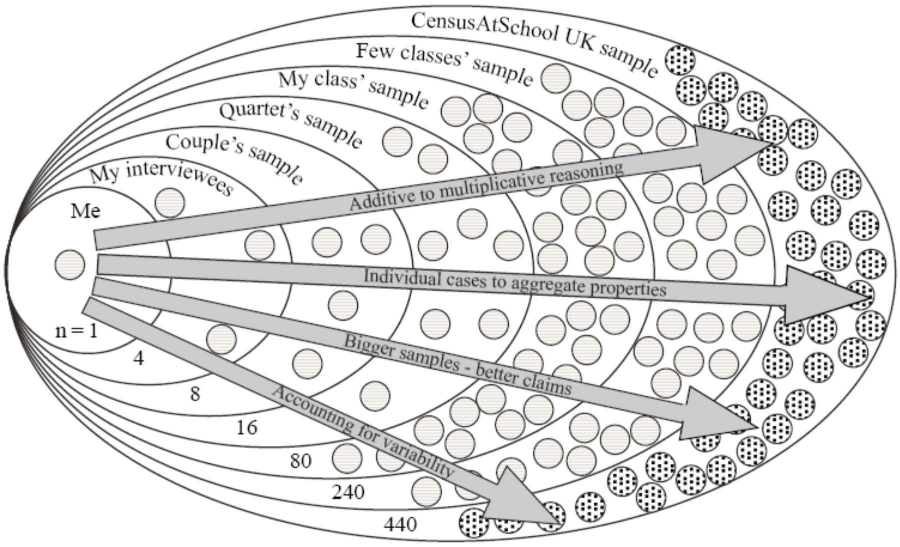

Figure 2: The growing samples sequence and the change in students' reasoning about informal inference.

PRELIMINARY RESULTS

The preliminary analysis of data sheds light on some of the ways fifth graders started to develop views of informal inference which are somehow consistent with those of statistics experts. The data also provides rich resource for understanding students' difficulties, informal ideas, argumentation and approaches as they begin to informally negotiate the notion of statistical inference.

Most students were able to produce reasonable inferences and arguments, meaningful to them and their classroom community. Students enjoyed their deep involvement in the challenging learning units, and worked seriously on all components of the project. Many of them appreciated the openness of the data exploration tasks, while others found it frustrating and needed more guidance. They used *TinkerPlots* fluently with a sense of enthusiasm. The full report will include ways in which the software supported as well as hindered students' reasoning about informal inference. For example, one clear advantage was students' use of the software as an argumentative tool in presenting their ideas to others instead of just a representation tool. On the other hand the emphasis on dots (representing cases) in *TinkerPlots* focused attention on individual views of data and hindered spontaneous progress in some cases to aggregate views of data.

The design of the learning trajectory, based on growing samples and the use of *Tinkerplots* helped students to develop aspects of informal inference and argumentative reasoning. We identified various levels of changes in students' statistical reasoning in multiple dimensions: Progress from additive reasoning to multiplicative reasoning, consideration of aggregate views of data, acknowledgment of the important role of larger samples, and accounting for variability (see arrows in Figure 2). The emergence of students' statistical knowledge was accompanied by growing ability to discuss their thoughts and actions, explain their inferences and argue about data-based claims.

Students firstly gained an intimate knowledge of small samples: They were able to personalize each case icon in a plot and even remembered many values by heart. This deep knowledge helped them in identifying missing or erroneous values, interpreting plots, and explaining irregularities and patterns in them. In addition, manipulation of these small samples and speculating and reasoning with stable features of variable processes served as a stepping stone to moving beyond focusing on individual cases to search for aggregate phenomena and to growing appreciation of the indispensability of larger samples. Some students however remained attached to individual view of data, which limited the scope of their inferences and quality of argumentation.

Interactions within the students group had an important role by leading to questioning and critiquing individual perspectives in a mutually supportive fashion so that a clearer understanding of informal inference emerged and knowledge of statistical ideas developed. Moreover, significant improvements were found in students' statistical knowledge, assessed by pre-post tests in statistics.

ICOTS-7 Presentation

The presentation will be mostly qualitative, demonstrating both the analysis and outcomes. We hope for an interactive discussion on three aspects of the study: a) the results in terms of what can be learned about primary students' negotiating meanings of inference in the context of increasing samples in statistical problem solving processes aided by *TinkerPlots*; b) the instructional implications of the study; and c) the suggestion of future research directions.

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