Chapter 1

STATISTICAL LITERACY, REASONING, AND THINKING: GOALS, DEFINITIONS, AND CHALLENGES

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INTRODUCTION

Over the past decade there has been an increasingly strong call for statistics education to focus more on statistical literacy, reasoning, and thinking. One of the main arguments presented is that traditional approaches to teaching statistics focus on skills, procedures, and computations, which do not lead students to reason or think statistically. This book explores the challenge posed to educators at all levels—how to develop the desired learning goals for students by focusing on current research studies that examine the nature and development of statistical literacy, reasoning, and thinking. We begin this introductory chapter with an overview of the reform movement in statistics education that has led to the focus on these learning outcomes. Next, we offer some preliminary definitions and distinctions for these often poorly defined and overlapping terms. We then describe some of the unique issues addressed by each chapter and conclude with some summary comments and implications.

THE GROWING IMPORTANCE OF STATISTICS IN TODAY'S WORLD

Quantitative information is everywhere, and statistics are increasingly presented as a way to add credibility to advertisements, arguments, or advice. Being able to properly evaluate evidence (data) and claims based on data is an important skill that all students should learn as part of their educational programs. The study of statistics provides tools that informed citizens need in order to react intelligently to quantitative information in the world around them. Yet many research studies indicate that adults in mainstream society cannot think statistically about important issues that affect their lives.

As former president of the American Statistical Association, David Moore (1990) wrote, "Statistics has some claim to being a fundamental method of inquiry, a general way of thinking that is more important than any of the specific techniques that make up the discipline" (p. 134). It is not surprising, given the importance of statistics, that there has been an increase in the amount of statistical content included in the elementary and secondary mathematics curriculum (NCTM, 2000) and an ever-increasing number of introductory statistics courses taught at the college level.

THE CHALLENGE OF TEACHING STATISTICS

Despite the increasing need for statistics instruction, historically statistics education has been viewed by many students as difficult and unpleasant to learn, and by many instructors as frustrating and unrewarding to teach. As more and more students enroll in introductory statistics courses, instructors are faced with many challenges in helping these students succeed in the course and learn statistics. Some of these challenges include

- Many statistical ideas and rules are complex, difficult, and/or counterintuitive. It is difficult to motivate students to engage in the hard work of learning statistics.
- Many students have difficulty with the underlying mathematics (such as fractions, decimals, algebraic formulas), and that interferes with learning the related statistical content.
- The context in many statistical problems may mislead the students, causing them to rely on their experiences and often faulty intuitions to produce an answer, rather than select an appropriate statistical procedure.
- Students equate statistics with mathematics and expect the focus to be on numbers, computations, formulas, and one right answer. They are uncomfortable with the messiness of data, the different possible interpretations based on different assumptions, and the extensive use of writing and communication skills.

Amidst the challenges of dealing with students' poor mathematics skills, low motivation to learn a difficult subject, expectations about what the course should be, and reliance on faulty intuitions and misconceptions, many instructors strive to enable students to develop statistical literacy, reasoning, and thinking. There appears to be a consensus that these are the most important goals for students enrolled in statistics classes, and that these goals are not currently being achieved. The dissatisfaction with students' ability to think and reason statistically, even after formally studying statistics at the college and graduate level, has led to a reexamination of the field of statistics.

EFFORTS TO CHANGE THE TEACHING OF STATISTICS

Today's leading statisticians see statistics as a distinct discipline, and one that is separate from mathematics (see Chapter 4). Some suggest that statistics should in fact be considered one of the liberal arts (e.g., Moore, 1998). The liberal arts image emphasizes that statistics involves distinctive and powerful ways of thinking: "Statistics is a general intellectual method that applies wherever data, variation, and chance appear. It is a fundamental method because data, variation, and chance are omnipresent in modern life. It is an independent discipline with its own core ideas rather than, for example, a branch of mathematics" (Moore, 1998, p. 1254).

As the discipline has evolved and become more distinct, changes have been called for in the teaching of statistics. Dissatisfaction with the introductory college course has led to a reform movement that includes focusing statistics instruction more on data and less on theory (Cobb, 1992). Moore (1997) describes the reform in terms of changes in content (more data analysis, less probability), pedagogy (fewer lectures, more active learning), and technology (for data analysis and simulations).

At the elementary and secondary level, there is an effort to help students develop an understanding and familiarity with data analysis (see Chapter 6) rather than teaching them a set of separate skills and procedures. New K–12 curricular programs set ambitious goals for statistics education, including developing students' statistical reasoning and understanding (e.g., Australia—Australian Education Council, 1991, 1994; England—Department for Education and Employment, 1999; New Zealand—Ministry of Education, 1992; USA—National Council of Teachers for Mathematics, 2000; and Project 2061's Benchmarks for Science Literacy, American Association for the Advancement of Science, 1993).

Several factors have led to these current efforts to change the teaching of statistics at all educational levels. These factors include

- Changes in the field of statistics, including new techniques of data exploration
- Changes and increases in the use of technology in the practice of statistics, and its growing availability in schools and at home
- Increased awareness of students' inability to think or reason statistically, despite good performance in statistics courses
- Concerns about the preparation of teachers of statistics at the K–12 and college level, many of whom have never studied applied statistics nor engaged in data analysis activities.

Many recommendations have been given for how statistics courses should be taught, as part of the general reform movement. Some of these recommendations are as follows:

- Incorporate more data and concepts.
- Rely heavily on real (not merely realistic) data.
- Focus on developing statistical literacy, reasoning, and thinking.

- Wherever possible, automate computations and graphics by relying on technological tools.
- Foster active learning, through various alternatives to lecturing.
- Encourage a broader range of attitudes, including appreciation of the power of statistical processes, chance, randomness, and investigative rigor, and a propensity to become a critical evaluator of statistical claims.
- Use alternative assessment methods to better understand and document student learning.

There appears to have been some impact on teaching practices from these recommendations at the college level (Garfield, Hogg, Schau, & Whittinghill, 2002). However, despite reform efforts, many statistics courses still teach the same progression of content and emphasize the development of skills and procedures. Although students and instructors appear to be happier with reformed courses, many students still leave the course perceiving statistics as a set of tools and techniques that are soon forgotten. Pfannkuch and Wild (Chapter 2) discuss how current methods of teaching have often focused on the development of skills and have failed to instill the ability to think statistically.

STATISTICAL LITERACY, REASONING, AND THINKING: DEFINITIONS AND DISTINCTIONS

It is apparent, when reading articles about recommendations to reform the teaching of statistics, that there are no consistent definitions for the often stated learning goals of literacy, reasoning, and thinking. Statistical literacy is used interchangeably with quantitative literacy, while statistical thinking and reasoning are used to define the same capabilities. This confusion of terms was especially evident at the Fifth International Conference on Teaching Statistics, held in Singapore in 1998. It became apparent that when statistics educators or researchers talk about or assess statistical reasoning, thinking, or literacy, they may all be using different definitions and understandings of these cognitive processes.

The similarities and differences among these processes are important to consider when formulating learning goals for students, designing instructional activities, and evaluating learning by using appropriate assessment instruments. A small, focused conference consisting of researchers interested in these topics appeared to be an important next step in clarifying the issues, connecting researchers and their studies, and generating some common definitions, goals, and assessment procedures. The first International Research Forum on Statistical Reasoning, Thinking, and Literacy (SRTL-1) was held in Israel in 1999 to address these needs. At this first conference some preliminary definitions were presented and discussed. A second forum (SRTL-2) was held in 2001 in Australia, with a focus on different types of statistical reasoning. Many of the papers from these first two forums have led to chapters in this book. The forums continue to be offered every two years (SRTL-3 in USA, 2003) as interest and research in this area steadily increase.

Although no formal agreement has been made regarding the definitions and distinctions of statistical literacy, reasoning, and thinking, the following list summarizes our current thoughts (Garfield, delMas, & Chance, 2003):

- ***Statistical literacy*** includes basic and important skills that may be used in understanding statistical information or research results. These skills include being able to organize data, construct and display tables, and work with different representations of data. Statistical literacy also includes an understanding of concepts, vocabulary, and symbols, and includes an understanding of probability as a measure of uncertainty.
- ***Statistical reasoning*** may be defined as the way people reason with statistical ideas and make sense of statistical information. This involves making interpretations based on sets of data, representations of data, or statistical summaries of data. Statistical reasoning may involve connecting one concept to another (e.g., center and spread), or it may combine ideas about data and chance. Reasoning means understanding and being able to explain statistical processes and being able to fully interpret statistical results.
- ***Statistical thinking*** involves an understanding of why and how statistical investigations are conducted and the "big ideas" that underlie statistical investigations. These ideas include the omnipresent nature of variation and when and how to use appropriate methods of data analysis such as numerical summaries and visual displays of data. Statistical thinking involves an understanding of the nature of sampling, how we make inferences from samples to populations, and why designed experiments are needed in order to establish causation. It includes an understanding of how models are used to simulate random phenomena, how data are produced to estimate probabilities, and how, when, and why existing inferential tools can be used to aid an investigative process. Statistical thinking also includes being able to understand and utilize the context of a problem in forming investigations and drawing conclusions, and recognizing and understanding the entire process (from question posing to data collection to choosing analyses to testing assumptions, etc.). Finally, statistical thinkers are able to critique and evaluate results of a problem solved or a statistical study.

For more discussion of these definitions and distinction, see papers by Chance (2002), delMas (2002), Garfield (2002), Rumsey (2002), and Chapters 2 through 4 in this book.

RATIONALE AND GOALS FOR THIS BOOK

With the increasing attention given to the need to develop students' statistical literacy, reasoning, and thinking at all levels, it has become apparent that these educational outcomes were not being adequately addressed in the research literature and, therefore, not used as the foundation for curricular programs. In fact, research studies on statistical reasoning are still evolving, and are just beginning to suggest ways to help students develop these outcomes.

Our goal in creating this book is to provide a useful resource for educators and researchers interested in helping students at all educational levels to develop statistical literacy, statistical reasoning, and statistical thinking. Given the increased attention being paid worldwide to the need for statistically literate citizens, the broad inclusion of statistics in the K–12 mathematics curriculum, the increasing numbers of students taking statistics at the secondary level (e.g., Advanced Placement Statistics courses in high school in the USA), and the increasing numbers of students required to take introductory statistics courses in postsecondary programs, it is crucial that the cutting-edge research being conducted on teaching and learning statistics be collected and disseminated along with specific pedagogical implications.

This book offers a synthesis of an emerging field of study, while at the same time responding to clear practical needs in the following ways:

- It establishes a research base for statistics education by focusing on and distinguishing between different outcomes of statistics instruction.
- It raises awareness of unique issues related to teaching and learning statistics, and it distinguishes statistical literacy, reasoning, and thinking from both general and mathematical literacy, reasoning, and thinking.
- It provides a bridge between educational research and practice, by offering research-based guidelines and suggestions to educators and researchers.

Although the word *statistics* is often used to represent both probability and statistical analysis, the authors and editors of this book focus on reasoning and thinking exclusively on the statistical analysis area, rather than on probability. Although statistics as a discipline uses mathematics and probability, probability is actually a field of mathematics. Since most of the early work in statistics education focused on the teaching and learning of probability, we wanted to move away and look at how students come to reason and think about data and data analysis. However, because the two subjects are so interrelated, several chapters mention issues related to learning probability as they relate to the focus of a particular chapter.

AUDIENCE FOR THIS BOOK

This book was designed to appeal to a diverse group of readers. The primary audience for this book is current or future researchers in statistics education (e.g., graduate students). However, we encourage others who do not identify themselves as researchers to read the chapters in this book as a way to understand the current issues and challenges in teaching and learning statistics. By asking authors to specifically address implications for teaching and assessing students, we hope that teachers of students at all levels will find the research results directly applicable to working with students.

SUGGESTED WAYS TO USE THIS BOOK

Given the different audiences for this book, we suggest several different ways to use this book for researchers, teachers, curriculum writers, and technology developers.

- **Researchers** Each chapter includes a detailed review of the literature related to a particular topic (e.g., reasoning about variability, statistical literacy, statistics teachers' development), which will be helpful to researchers studying one of these areas. The chapters also provide examples of current research methodologies used in this area, and present implications for teaching practice as well as suggestions for future research studies. By providing cutting-edge research on statistical literacy, reasoning, and thinking, the book as a whole outlines the state of the art for the statistics education research community. In addition, the contributing authors may be regarded as useful human resources for researchers who are interested in pursuing studies in these areas.
- **Curriculum writers** By reading this book, people designing statistics instructional activities and curricula may learn about current research results in statistics education. Curriculum development involves tightly integrated cycles of reviewing related research, instructional design, and analysis of students' learning, which all feed back to inform the revision of the design. Many chapters in this book also give recommendations for appropriate ways to assess learning outcomes.
- **Technology** Many chapters in this book offer discussion on the role of technology in developing statistical reasoning. Types of technologies used are presented and assessed in relation to their impact on students' reasoning.

Given the different uses just listed, we believe that this book can be used in a variety of graduate courses. Such courses include those preparing mathematics teachers at the K–12 level; courses preparing teachers of statistics at the high secondary and tertiary level; and research seminars in mathematics, statistics education, or psychology.

We advise readers focused on students at one level (e.g., secondary) not to skip over chapters describing students at other levels. We are convinced that students who are introduced to statistical ideas and procedures learn much the same material and concepts (e.g., creating graphical displays of data, describing the center and dispersion of data, inference from data, etc.) regardless of their grade level. Furthermore, reasoning processes develop along extended periods of time, beginning at early encounters with data in elementary grades and continuing through high school and postsecondary education. Therefore, we believe that discussions of reasoning issues couched in the reality of one age group will be of interest to those working with students of other ages and abilities.

OVERVIEW OF CHAPTERS

All of the chapters in this book discuss issues pertaining to statistical literacy, reasoning, or thinking. Some chapters focus on general topics (e.g., statistical literacy) while others focus on the context of a specific educational level or setting (e.g., teaching middle school students to reason about distribution). Whenever possible, the chapter authors outline challenges facing educators, statisticians, and other stakeholders. The chapters present many examples (or references to resources) of activities, data sets, and assessment tasks suitable for a range of instructional levels. This emphasis of connection to practice is a result of our strong belief that researchers are responsible for translating their findings to practical settings.

All the chapters that focus on a particular type of student or teacher statistical reasoning (Chapters 6 through 15) follow a unified and familiar structure to facilitate their effective use by the readers. These chapters typically start with a section introducing the key area of reasoning explored in the chapter. This is followed by clear and informative descriptions of the *problem* (a description of the type of reasoning studied, why it is important, and how this type of reasoning fits into the curriculum); *literature and background* (prior and related work and relevant theoretical background); *methods* (the subjects, methods used, data gathered, and activities or interventions used); *analysis and results* (description of how the data were analyzed, and the results and findings of the study); and *discussion* (lessons learned from the study, new questions raised, limitations found). Finally, in the *implications* section, each chapter highlights key practical implications related to teaching and assessing students as well as implications for research.

The chapters have been grouped into three parts, each of which is summarized here.

Part I. Introduction to Statistical Literacy, Reasoning, and Thinking (Chapters 2 through 5)

The first part of this book is a comprehensive overview of the three interrelated but distinct cognitive processes (or learning outcomes) of statistical literacy, reasoning, and thinking from historical, psychological, and educational perspectives. This part is therefore the basis upon which the individual studies in subsequent parts are built.

In the first chapter of this part (Chapter 2), Pfannkuch and Wild present their paradigm on statistical thinking (part of their four-dimensional framework for statistical thinking in empirical enquiry; Wild & Pfannkuch, 1999). The authors identify five types of thinking, considered to be fundamental to statistics. They follow the origins of statistical thinking through to an explication of what is currently understood to be statistical thinking. They begin their historical exploration with the early developers of statistics; move on to more recent contributions from epidemiology, psychology, and quality management; and conclude with a discussion of recent writings of statistics education researchers and statisticians influential in the movement of pedagogy from methods toward thinking.

Gal proposes in Chapter 3 a conceptualization of statistical literacy and its main components. Statistical literacy is described as a key ability expected of citizens in information-laden societies, an expected outcome of schooling, and a necessary component of adults' numeracy and literacy. Statistical literacy is portrayed as the ability to interpret, critically evaluate, and communicate about statistical information and messages. Gal suggests that statistically literate behavior is predicated on the joint activation of both a *knowledge* component (comprised of five cognitive elements: literacy skills, statistical knowledge, mathematical knowledge, context knowledge, and critical questions) and a *dispositional* component (comprised of two elements: critical stance, and beliefs and attitudes).

The focus of delMas's chapter (Chapter 4) is on the nature of mathematical and statistical reasoning. The author first outlines the general nature of human reasoning, which he follows with an account of mathematical reasoning as described by mathematicians along with recommendations by mathematics educators regarding educational experiences to improve mathematical reasoning. He reviews the literature on statistical reasoning and uses findings from the general literature on reasoning to identify areas of statistical reasoning that students find most challenging. Finally, he compares and contrasts statistical reasoning and mathematical reasoning.

The last chapter in this part (Chapter 5) is a joint work by Jones, Langrall, Mooney, and Thornton that examines cognitive models of development in statistical reasoning and the role they can play in statistical education. The authors consider models of development from a psychological perspective, and then describe how models of statistical reasoning have evolved historically from models of development in probability. The authors describe and analyze comprehensive models of cognitive development that deal with multiple processes in statistical reasoning as well as models of cognitive development that characterize students' statistical reasoning as they deal with specific areas of statistics and data exploration. The authors suggest that school students' statistical reasoning passes through a number of hierarchical levels and cycles.

Part II. Studies of Statistical Reasoning (Chapters 6 through 13)

The chapters in this part focus on how students reason about specific areas of statistics. The topics of these chapters include data analysis, distributions, measures of center, variation, covariation, normal distribution, samples, and sampling distributions. These studies represent the current efforts in the statistics education community to focus statistical instruction and research on the big ideas of statistics (Chapter 17) and on developing students' statistical reasoning at all levels of education.

In the first chapter of this part (Chapter 6), Ben-Zvi describes and analyzes the ways in which middle school students begin to reason about data and come to understand exploratory data analysis (EDA). He describes the process of developing reasoning about data while learning skills, procedures, and concepts. In addition, the author observes the students as they begin to adopt and exercise some of the habits and points of view that are associated with statistical thinking. Ben-Zvi offers two case studies focusing on the development of a global view of data and data representations, and on design of a meaningful EDA learning environment that promotes statistical reasoning about data analysis. In light of the analysis, the author proposes a description of what it may mean to learn to reason about data analysis.

Bakker and Gravemeijer explore (Chapter 7) how informal reasoning about distribution can be developed in a technological learning environment. They describe the development of reasoning about distribution in seventh-grade classes in three stages as students reason about different representations. The authors show how specially designed software tools, students' created graphs, and prediction tasks supported the learning of different aspects of distribution. In this process, several students came to reason about the shape of a distribution using the term *bump* along with statistical notions such as outliers and sample size.

Chapter 8 presents an article by Konold and Pollatsek originally published in a research journal; therefore, it does not follow the same format as the other chapters in this part. Their chapter offers a conceptualization of averages as a stable feature of a noisy process. To explore the challenges of learning to think about data as signal and noise, the authors examine that metaphor in the context of three different types of statistical processes. For each process, they evaluate the conceptual difficulty of regarding data from that process as a combination of signal and noise. The authors contrast this interpretation of averages with various other interpretations of averages (e.g., summaries of groups of values) that are frequently encountered in curriculum materials. They offer several recommendations about how to develop and extend the idea of central tendency as well as possible directions for research on student thinking and learning.

Understanding the nature of variability and its omnipresence is a fundamental component of statistical reasoning. In Chapter 9, Reading and Shaughnessy bring together findings from a number of different studies, conducted in three different countries, designed to investigate students' conceptions of variability. The focus of the chapter is on details of one recent study that investigates reasoning about variation in a sampling situation for students aged 9 to 18.

In Chapter 10, Moritz investigates three skills of reasoning about covariation: (a) speculative data generation, demonstrated by drawing a graph to represent a verbal statement of covariation; (b) verbal graph interpretation, demonstrated by describing a scatterplot in a verbal statement and by judging a given statement; and (c) numerical graph interpretation, demonstrated by reading a value and interpolating a value. The authors describe survey responses from students in grades 3, 5, 7, and 9 in four levels of reasoning about covariation.

Batanero, Tauber, and Sánchez present (Chapter 11) the results of a study on students' learning of the normal distribution in a computer-assisted, university-level introductory course. The authors suggest a classification of different aspects of students' correct and incorrect reasoning about the normal distribution as well as giving examples of students' reasoning in the different categories.

Chapter 12, written by Watson, extends previous research on students' reasoning about samples and sampling by considering longitudinal interviews with students 3 or 4 years after they first discussed their understanding of what a sample was, how samples should be collected, and the representing power of a sample based on its size. Of the six categories of response observed at the time of the initial interviews, all were confirmed after 3 or 4 years, and one additional preliminary level was observed.

Reasoning about sampling distributions is the focus of Chance, delMas, and Garfield in the last chapter of this part (Chapter 13). In this chapter, the authors present a series of research studies focused on the difficulties students experience when learning about sampling distributions. In particular, the authors trace the 7-year history of an ongoing collaborative classroom-based research project investigating the impact of students' interaction with computer software tools to improve their reasoning about sampling distributions. The authors describe the complexities involved in building a deep understanding of sampling distributions, and formulate models to explain the development of students' reasoning.

Part III. Curricular, Instructional, and Research Issues (Chapters 14 through 16)

The third and final part of this book deals with important educational issues related to the development of students' statistical reasoning: (a) teachers' knowledge and understanding of statistics, and (b) instructional design issues.

Mickelson and Heaton (Chapter 14) explore the complexity of teaching and learning statistics, and offer insight into the role and interplay of teachers' statistical knowledge and context. Their study presents an analysis of one third-grade teacher's statistical reasoning about data and distribution in the context of classroom-based statistical investigation. In this context, the teacher's statistical reasoning plays a central role in the planning and orchestration of the class investigation.

Makar and Confrey also discuss (Chapter 15) teachers' statistical reasoning. They focus on the statistical reasoning about comparing two distributions of four secondary teachers addressing the research question: "How do you decide whether two groups are different?" The study was conducted at the end of a 6-month professional development sequence designed to assist secondary teachers in making sense of their students' results on a state-mandated academic test. The authors provide qualitative and quantitative analyses to examine the teachers' reasoning.

In Chapter 16, Cobb and McClain propose design principles for developing statistical reasoning about data in the contexts of EDA and data generation in elementary school. They present a short overview of a classroom design experiment, and then frame it as a paradigm case in which to tease out design principles addressing five aspects of the classroom environment that proved critical in supporting the students' statistical learning: The focus on central statistical ideas, the instructional activities, the classroom activity structure, the computer-based tools the students used, and the classroom discourse.

Summary and Implications (Chapter 17)

In the closing chapter (Chapter 17) the editors summarize issues, challenges, and implications for teaching and assessing students emerging from the collection of studies in this book. We begin with some comments on statistics education as an emerging research area, and then concentrate on the need to focus research, instruction, and assessment on the big ideas of statistics. We address the role of technology in developing statistical reasoning as well as the diversity of various statistics learners (e.g., students at different educational levels as well as their teachers). Next we present a summary of research methodologies used to study statistical reasoning, along with comments on the extensive use of qualitative methods and the lack of traditional experimental designs. Finally, we consider some implications for teaching and assessing students and suggest future research directions.

We hope that the articulated, coherent body of knowledge on statistical literacy, reasoning, and thinking presented in this book will contribute to the pedagogical effectiveness of statistics teachers and educators at all levels; to the expansion of research studies on statistical literacy, reasoning and thinking; and to growth of the statistics education community.

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